COMP9313: Big Data Management

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Course web site: http://www.cse.unsw.edu.au/~cs9313/

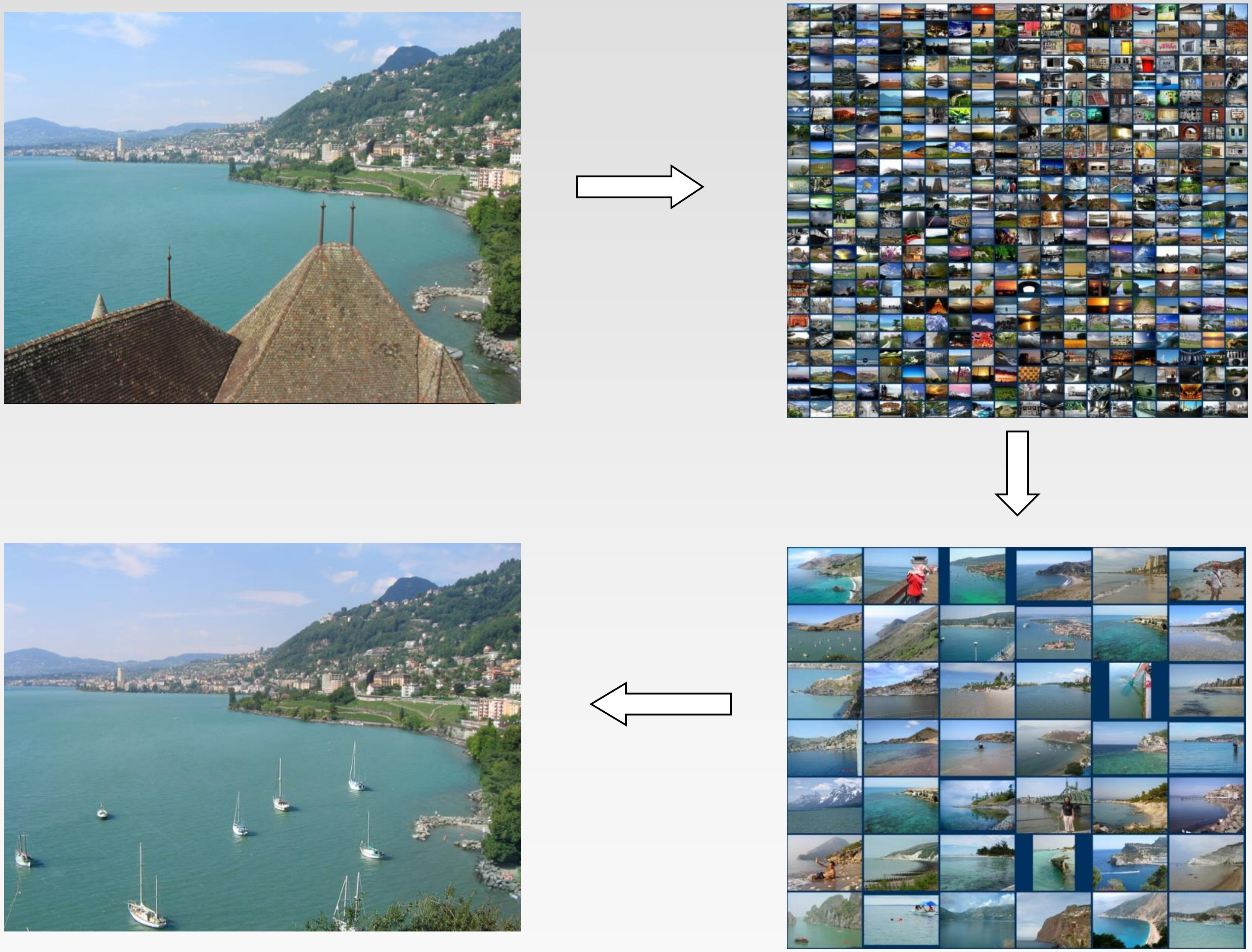
Chapter 8.1: Finding Similar Items

A Common Metaphor

- **Many problems can be expressed as finding "similar" sets:**
  - **Find near-neighbors in high-dimensional space**

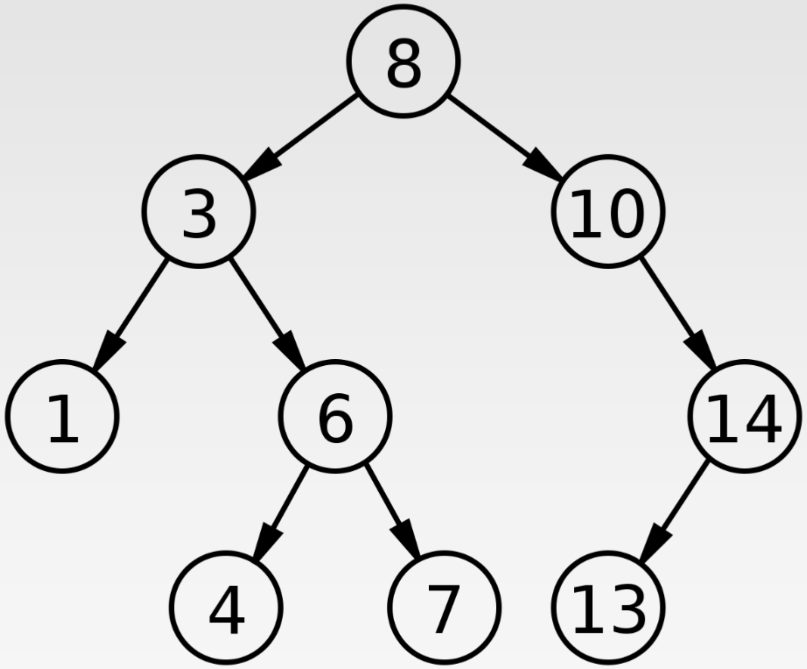
- **Examples:**
  - **Pages with similar words**
    - For duplicate detection, classification by topic
  - **Customers who purchased similar products**
    - Products with similar customer sets
  - **Images with similar features**
    - Google image search

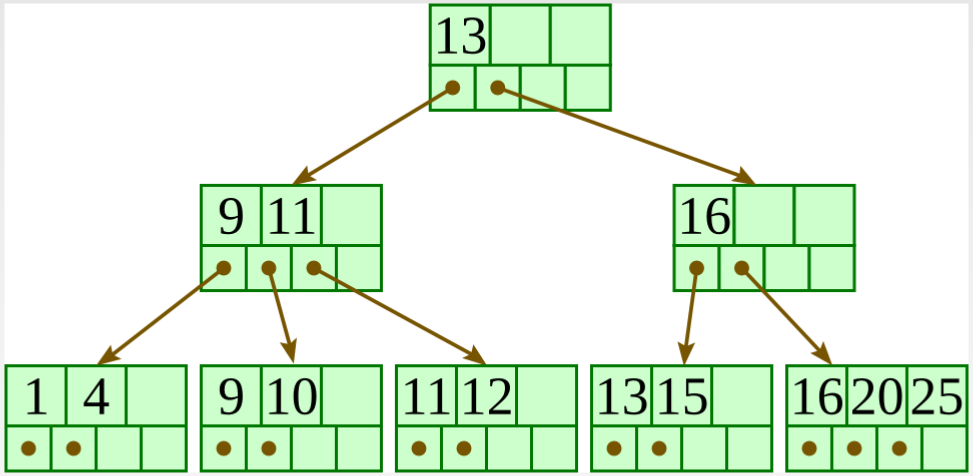
Images with Similar Features



Similarity Search in One Dimensional Space

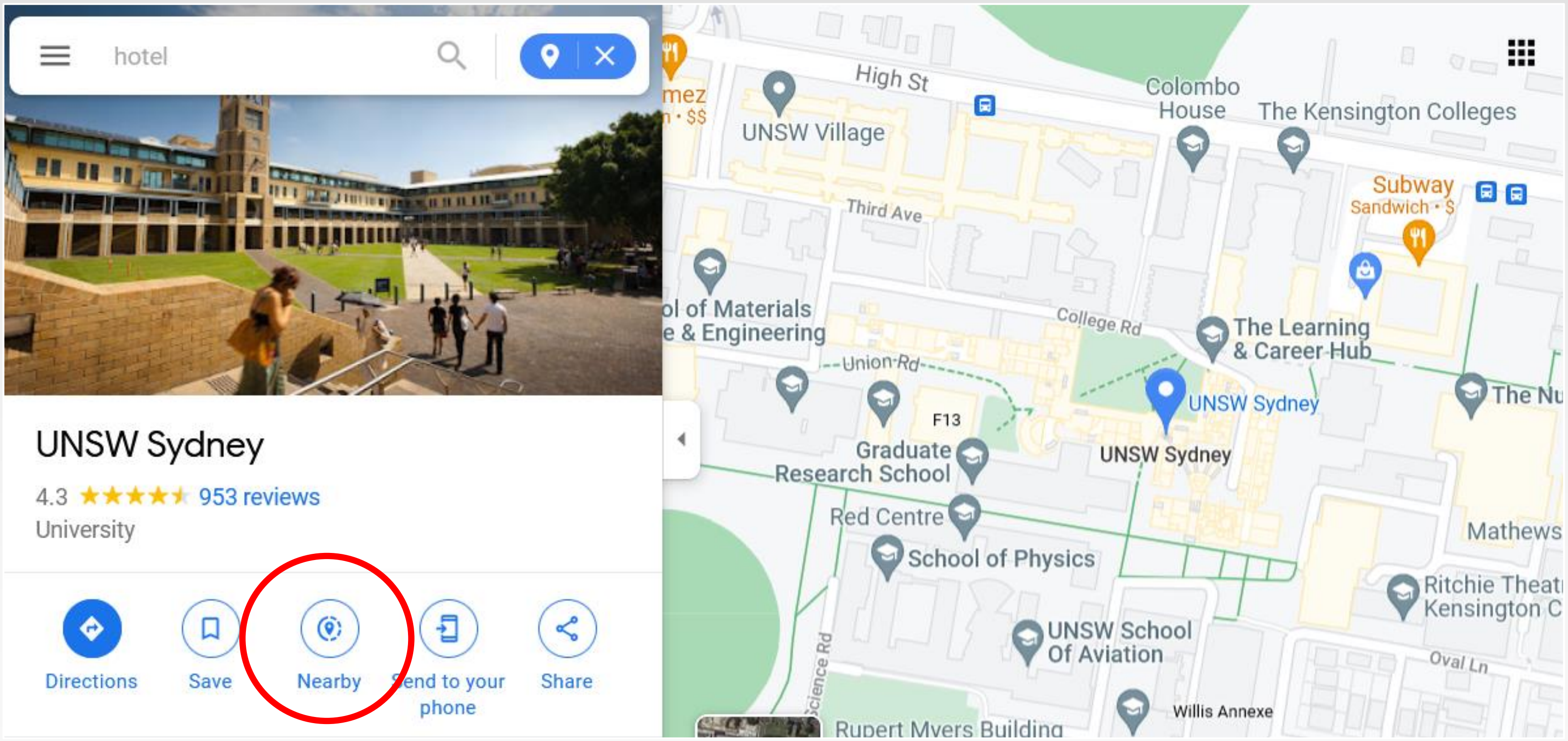
- Just numbers, use binary search, binary search tree, B+-Tree…
- The essential idea behind: objects can be sorted





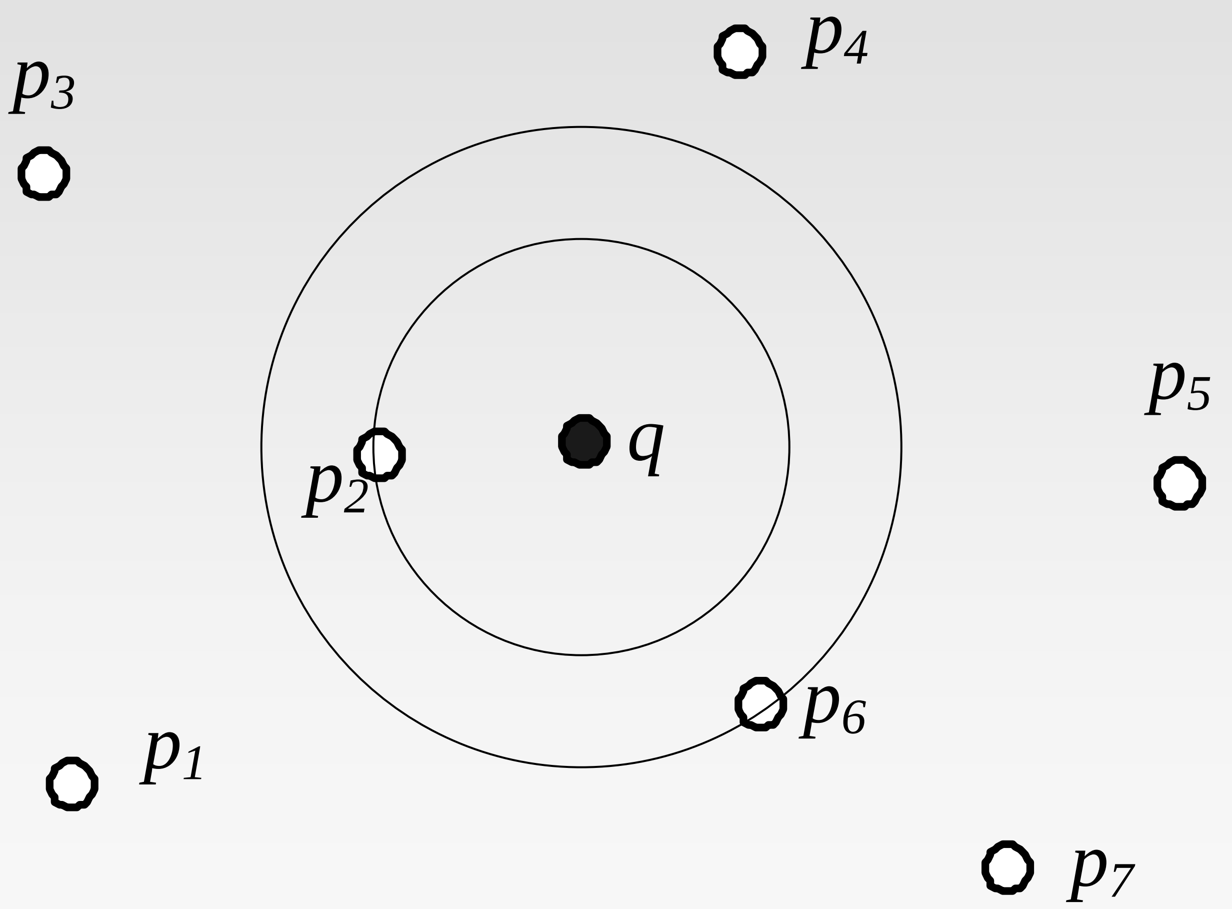
Similarity Search in 2D Space

- *k* nearest neighbour (*k*NN) query: find the top-k nearest spatial object to the query location
- E.g., find the top-5 closest restaurants to UNSW



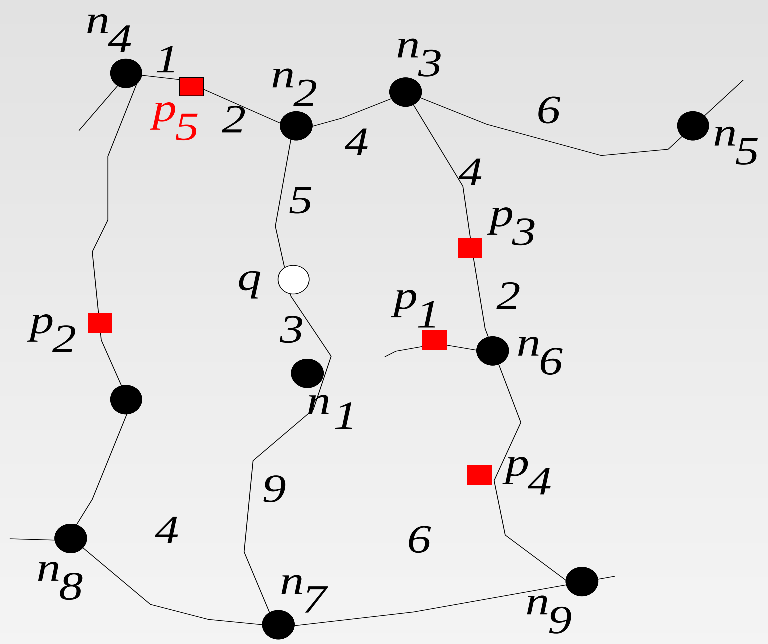
Similarity Search in 2D Space

- In Euclidean Space



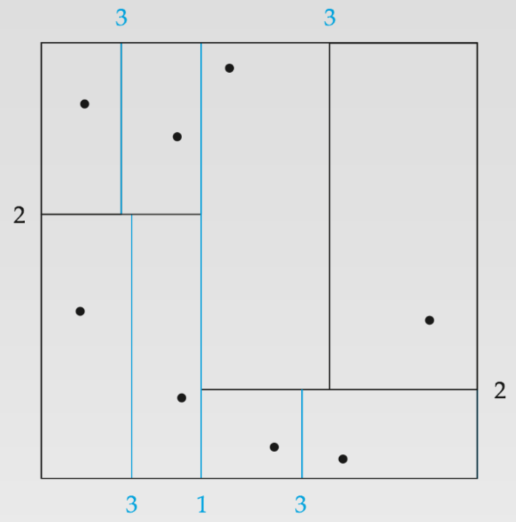
Similarity Search in 2D Space

- In road networks: Distance is computed based on the network distance (such as the length of the shortest path)

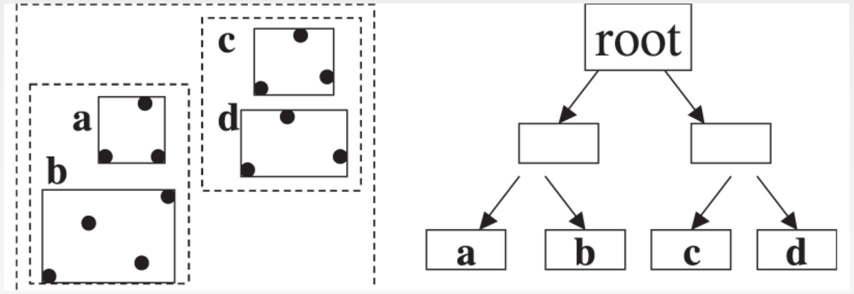


p_5 is the closest in the spatial network setting
p_1 is the closest in the Euclidean space

The Problem in 2D Space

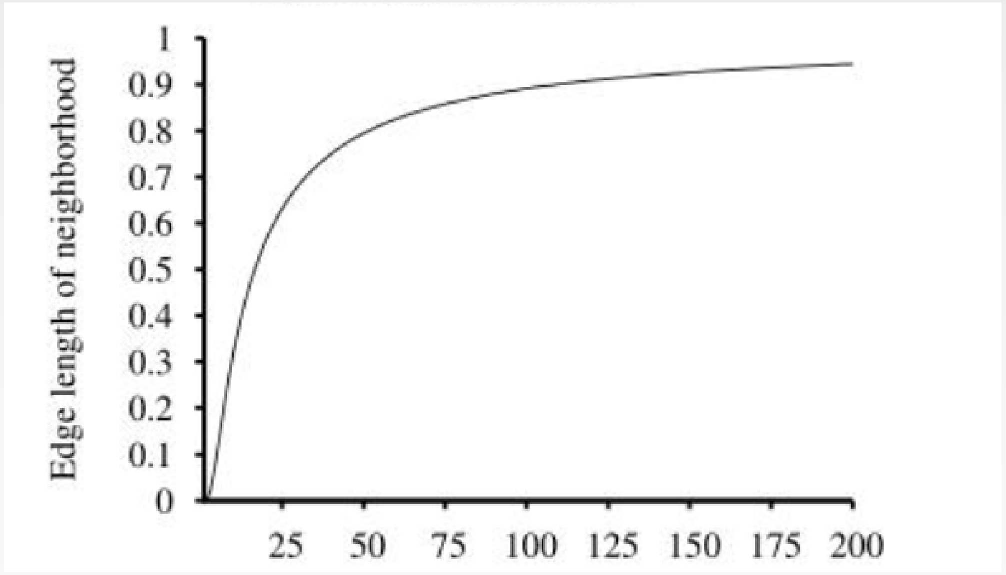


- Euclidean space
  - Grid index
  - Quad-tree
  - *k-d* tree
  - R-tree (R+-tree, R*-tree, etc.)
  - m-tree, x-tree, … …
  - Space filing curves: Z-order, Hilbert order, … …
- Road Networks
  - G-tree
  - Contraction Hierarchy
  - 2-hop labeling
  - … …



Curse of Dimensionality

- Refers to various phenomena that arise in high dimensional spaces that do not occur in low dimensional settings.
- Specifically, refers to the decrease in performance of similarity search query processing when the dimensionality increases.
- In high dimensional space, almost all points are far away from each other.
  - To find the top-10 nearest neighbors, what is the length of the average neighborhood cube?



Problem

- **Given: High dimensional data points $x_1, x_2, \ldots$**
  - **For example:** Image is a long vector of pixel colors

$$\begin{bmatrix} 1 & 2 & 1 \\ 0 & 2 & 1 \\ 0 & 1 & 0 \end{bmatrix} \rightarrow [1\ 2\ 1\ 0\ 2\ 1\ 0\ 1\ 0]$$

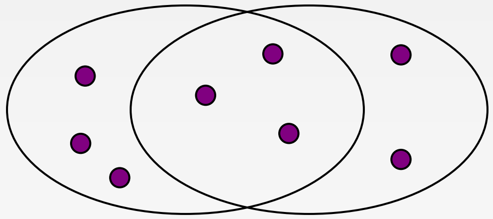
- **And some distance function $d(x_1, x_2)$**
  - Which quantifies the "distance" between x_1 and x_2
- **Goal:** Find **all pairs of data points** (x_i, x_j) that are within some distance threshold $d(x_i, x_j) \le s$
- **Note:** Naïve solution would take $O(N^2)$ ☹

  where N is the number of data points

- **MAGIC: This can be done in $O(N)$!! How?**

Distance Measures

- **Goal**: Find near-neighbors in high-dim. space
  - We formally define "near neighbors" as points that are a "small distance" apart
- For each application, we first need to define what "**distance**" means
- **Today: Jaccard distance/similarity**
  - The **Jaccard similarity** of two **sets** is the size of their intersection divided by the size of their union:
    $\boldsymbol{sim(C_1, C_2) = |C_1 \cap C_2| / |C_1 \cup C_2|}$
  - **Jaccard distance:** $\boldsymbol{d(C_1, C_2) = 1 - |C_1 \cap C_2| / |C_1 \cup C_2|}$



3 in intersection
8 in union
Jaccard similarity= 3/8
Jaccard distance = 5/8

Task: Finding Similar Documents

- **Goal: Given a large number (N in the millions or billions) of documents, find "near duplicate" pairs·**
- **Applications:**
  - Mirror websites, or approximate mirrors
    - Don't want to show both in search results
  - Similar news articles at many news sites
    - Cluster articles by "same story"
- **Problems:**
  - Many small pieces of one document can appear out of order in another
  - Too many documents to compare all pairs
  - Documents are so large or so many that they cannot fit in main memory

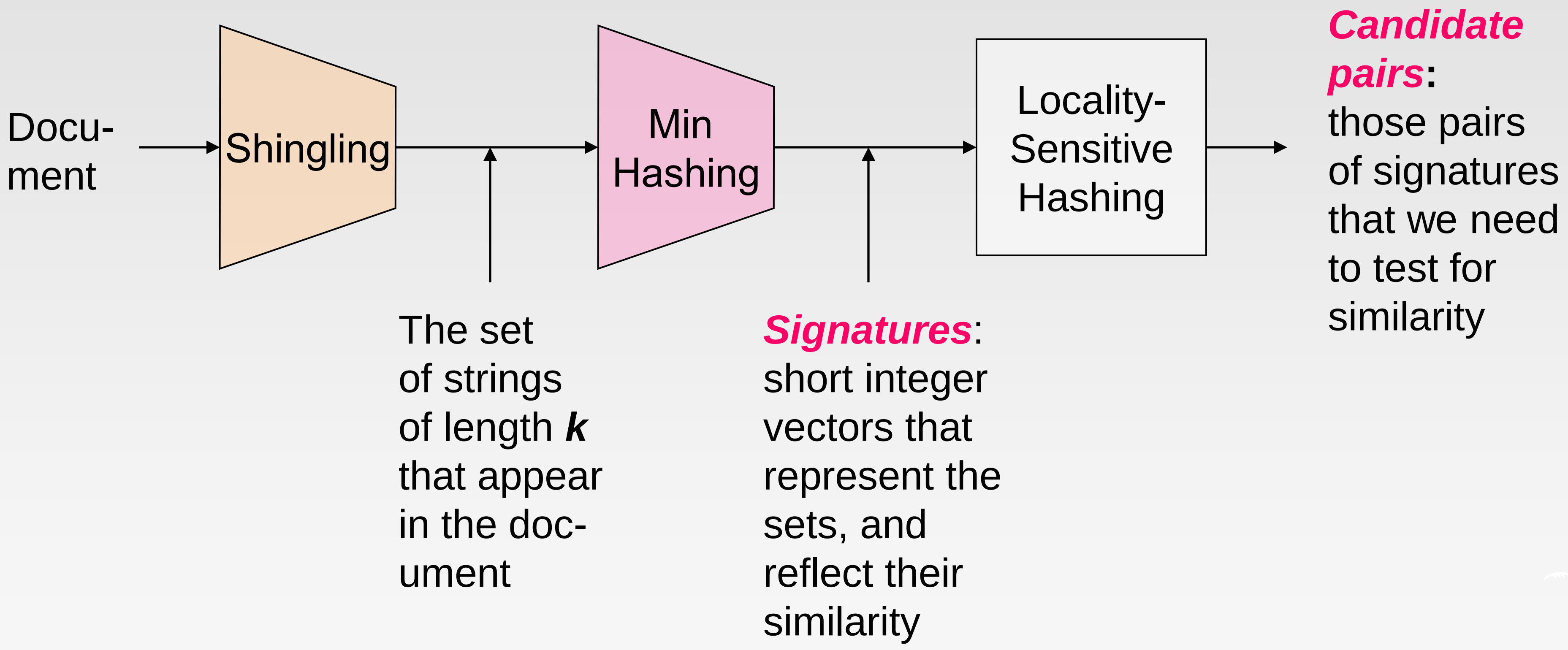
3 Essential Steps for Similar Docs

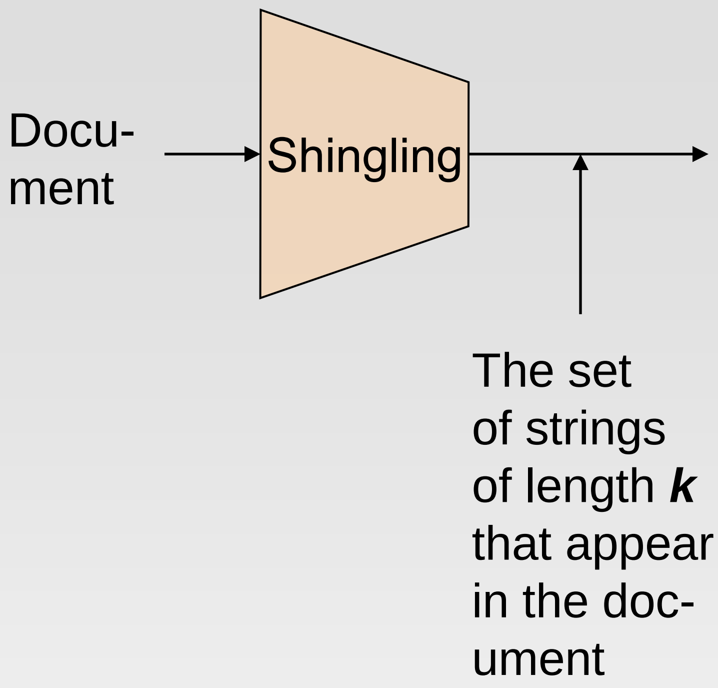
1. ***Shingling:*** Convert documents to sets

2. ***Min-Hashing:*** Convert large sets to short signatures, while preserving similarity

3. ***Locality-Sensitive Hashing:*** Focus on pairs of signatures likely to be from similar documents
   - **Candidate pairs!**

The Big Picture





Step 1: ***Shingling:*** Convert documents to sets

Documents as High-Dim Data

- **Step 1: *Shingling:* Convert documents to sets**

- **Simple approaches:**
  - Document = set of words appearing in document
  - Document = set of "important" words
  - Don't work well for this application. Why?

- **Need to account for ordering of words!**
- A different way: **Shingles!**

Define: Shingles

- A *k*-shingle (or *k*-gram) for a document is a sequence of *k* tokens that appears in the doc
  - Tokens can be characters, words or something else, depending on the application
  - Assume tokens = characters for examples

- **Example:** **k=2**; document **D_1** = abcab
  Set of 2-shingles: **$S(D_1)$** = {ab, bc, ca}
  - **Option:** Shingles as a bag (multiset), count ab twice: **$S'(D_1)$ =** {ab, bc, ca, ab}

Shingles and Similarity

- Documents that are intuitively similar will have many shingles in common.
- Changing a word only affects k-shingles within distance k-1 from the word.
- Reordering paragraphs only affects the 2k shingles that cross paragraph boundaries.
- Example: k=3, "The dog which chased the cat" versus "The dog that chased the cat".
  - Only 3-shingles replaced are g_w, _wh, whi, hic, ich, ch_, and h_c.

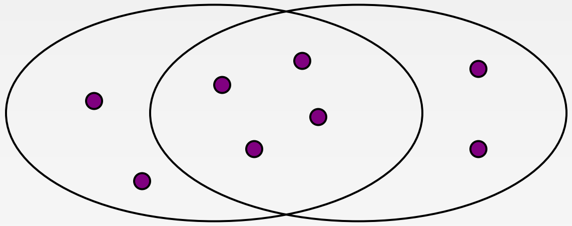
Compressing Shingles

- To **compress long shingles**, we can **hash** them to (say) 4 bytes
- **Represent a document by the set of hash values of its *k*-shingles**
  - **Idea:** Two documents could (rarely) appear to have shingles in common, when in fact only the hash-values were shared
- **Example:** **k=2**; document $\mathbf{D_1}$= abcab
  Set of 2-shingles: $\mathbf{S(D_1)}$ = {ab, bc, ca}
  Hash the singles: $\mathbf{h(D_1)}$ = {1, 5, 7}

Similarity Metric for Shingles

- **Document D_1 is a set of its k-shingles $C_1=S(D_1)$**
- Equivalently, each document is a
  0/1 vector in the space of *k*-shingles
  - Each unique shingle is a dimension
  - Vectors are very sparse
- **A natural similarity measure is the Jaccard similarity:**

$$sim(D_1, D_2) = |C_1 \cap C_2| / |C_1 \cup C_2|$$



Working Assumption

- **Documents that have lots of shingles in common have similar text, even if the text appears in different order**

- If we pick ***k*** too small, then we would expect most sequences of ***k*** characters to appear in most documents
  - We could have documents whose shingle-sets had high Jaccard similarity, yet the documents had none of the same sentences or even phrases
  - Extreme case: when we use ***k*** = 1, almost all Web pages will have high similarity.

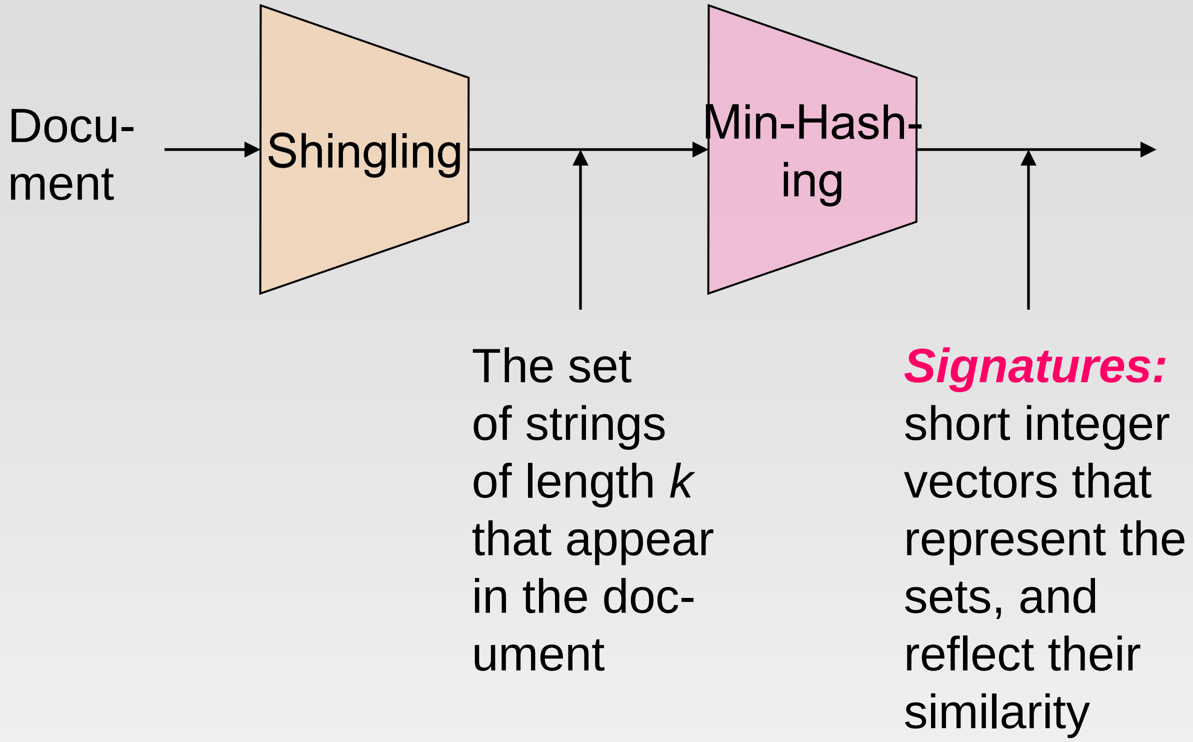
- **Caveat:** You must pick ***k*** large enough, or most documents will have most shingles
  - ***k*** = 5 is OK for short documents
  - ***k*** = 10 is better for long documents

Motivation for Minhash/LSH

- **Suppose we need to find near-duplicate documents among $N = 1$ million documents**

- Naïvely, we would have to compute **pairwise Jaccard similarities** for **every pair of docs**
  - $\boldsymbol{N(N-1)/2} \approx$ **5*10^{11}** comparisons
  - At 10^5 secs/day and 10^6 comparisons/sec, it would take **5 days**

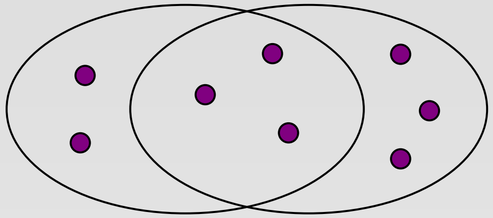
- For $\boldsymbol{N = 10}$ million, it takes more than a year…



Step 2: ***Minhashing:*** Convert **large sets** to **short signatures**, while **<u>preserving similarity</u>**

Encoding Sets as Bit Vectors

- Many similarity problems can be formalized as **finding subsets that have significant intersection**



- **Encode sets using 0/1 (bit, boolean) vectors**
  - One dimension per element in the universal set

- Interpret set intersection as bitwise **AND**, and set union as bitwise **OR**

- **Example:** $\mathbf{C_1}$ = 10111; $\mathbf{C_2}$ = 10011
  - Size of intersection **= 3**; size of union **= 4**,
  - **Jaccard similarity** (not distance) **= 3/4**
  - **Distance: $d(C_1,C_2)$ = 1 – (Jaccard similarity) = 1/4**

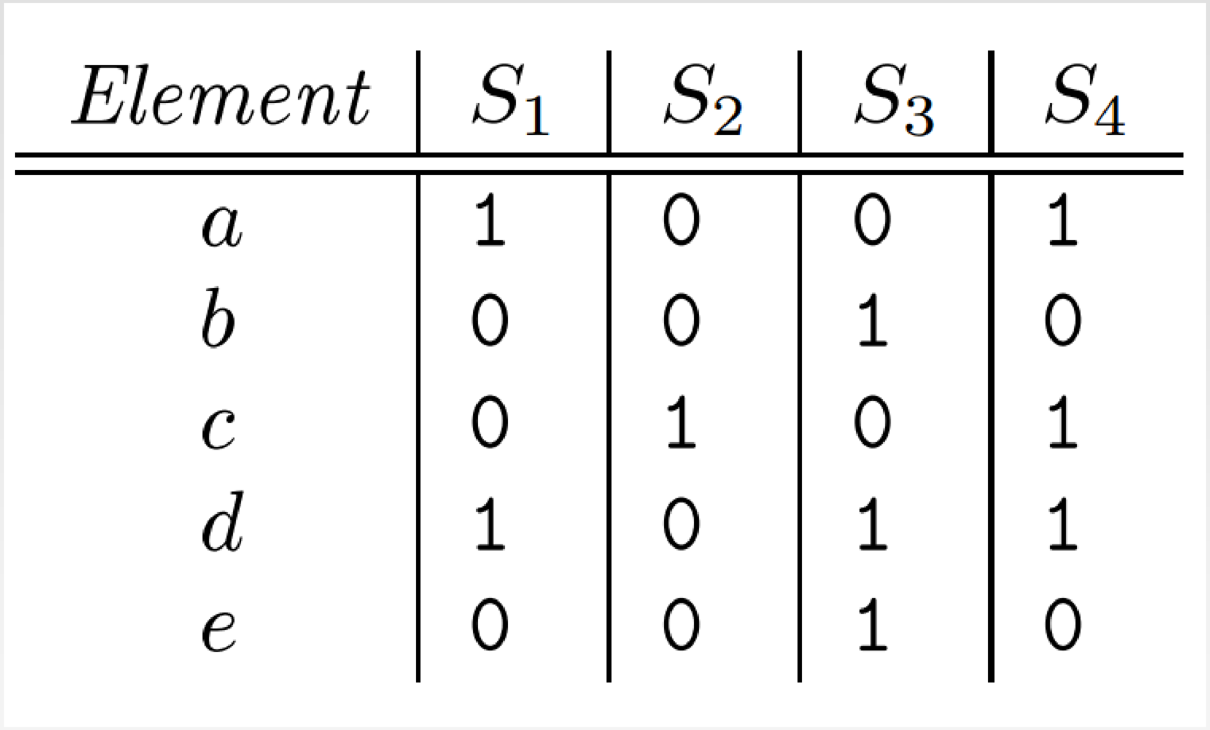
From Sets to Boolean Matrices

- **Rows** = elements (shingles)
- **Columns** = sets (documents)
  - 1 in row ***e*** and column ***s*** if and only if ***e*** is a member of ***s***
  - Column similarity is the Jaccard similarity of the corresponding sets (rows with value *1)*
  - **Typical matrix is sparse!**
- **Each document is a column:**

$$
\begin{array}{c|c|c|c|c|}
 & \multicolumn{4}{c}{\text{Documents}} \\
\hline
 & 1 & 1 & 1 & 0 \\
\hline
 & 1 & 1 & 0 & 1 \\
\hline
 & 0 & 1 & 0 & 1 \\
\hline
\text{Shingles} & 0 & 0 & 0 & 1 \\
\hline
 & 1 & 0 & 0 & 1 \\
\hline
 & 1 & 1 & 1 & 0 \\
\hline
 & 1 & 0 & 1 & 0 \\
\hline
\end{array}
$$

From Sets to Boolean Matrices

- **Example:** S_1 = {a, d}, S_2 = {c}, S_3 = {b, d, e}, and S_4 = {a, c, d}



Element	S_1	S_2	S_3	S_4
a	1	0	0	1
b	0	0	1	0
c	0	1	0	1
d	1	0	1	1
e	0	0	1	0

- **sim(S_1, S_3) = ?**
  - Size of intersection = 1; size of union = 4, Jaccard similarity (not distance) = 1/4
  - **d(S_1, S_3) = 1 – (Jaccard similarity) = 3/4**

Outline: Finding Similar Columns

- **So far:**
  - Documents → Sets of shingles
  - Represent sets as boolean vectors in a matrix
- **Next goal: Find similar columns while computing small signatures**
  - **Similarity of columns == similarity of signatures**

Outline: Finding Similar Columns

- **Next Goal: Find similar columns, Small signatures**
- **Naïve approach:**
  - **1) Signatures of columns:** small summaries of columns
  - **2) Examine pairs of signatures** to find similar columns
    - **Essential:** Similarities of signatures and columns are related
  - **3) Optional:** Check that columns with similar signatures are really similar
- **Warnings:**
  - Comparing all pairs may take too much time: **Job for LSH**
    - These methods can produce false negatives, and even false positives (if the optional check is not made)

Hashing Columns (Signatures)

- **Key idea:** "hash" each column ***C*** to a small ***signature*** ***h(C)***, such that:
  - **(1)** ***h(C)*** is small enough that the signature fits in RAM
  - **(2)** $sim(C_1, C_2)$ is the same as the "similarity" of signatures $h(C_1)$ and $h(C_2)$

- **Goal: Find a hash function *h(·)* such that:**
  - If $sim(C_1,C_2)$ is high, then with high prob. $h(C_1) = h(C_2)$
  - If $sim(C_1,C_2)$ is low, then with high prob. $h(C_1) \neq h(C_2)$

- **Hash docs into buckets. Expect that "most" pairs of near duplicate docs hash into the same bucket!**

Min-Hashing

- **Goal: Find a hash function *h(·)* such that:**
  - if ***sim*(C_1,C_2)** is high, then with high prob. ***h*(C_1) = *h*(C_2)**
  - if ***sim*(C_1,C_2)** is low, then with high prob. ***h*(C_1) ≠ *h*(C_2)**
- **Clearly, the hash function depends on the similarity metric:**
  - Not all similarity metrics have a suitable hash function
- **There is a suitable hash function for the Jaccard similarity: Min-Hashing**

Min-Hashing

- Imagine the rows of the boolean matrix permuted under **random permutation** π

- Define a **"hash" function** $h_\pi(C)$ = the index of the **first** (in the permuted order π) row in which column ***C*** has value **1**:

$$h_\pi(C) = \min_\pi \pi(C)$$

- Use several (e.g., 100) independent hash functions (that is, permutations) to create a signature of a column

Min-Hashing Example

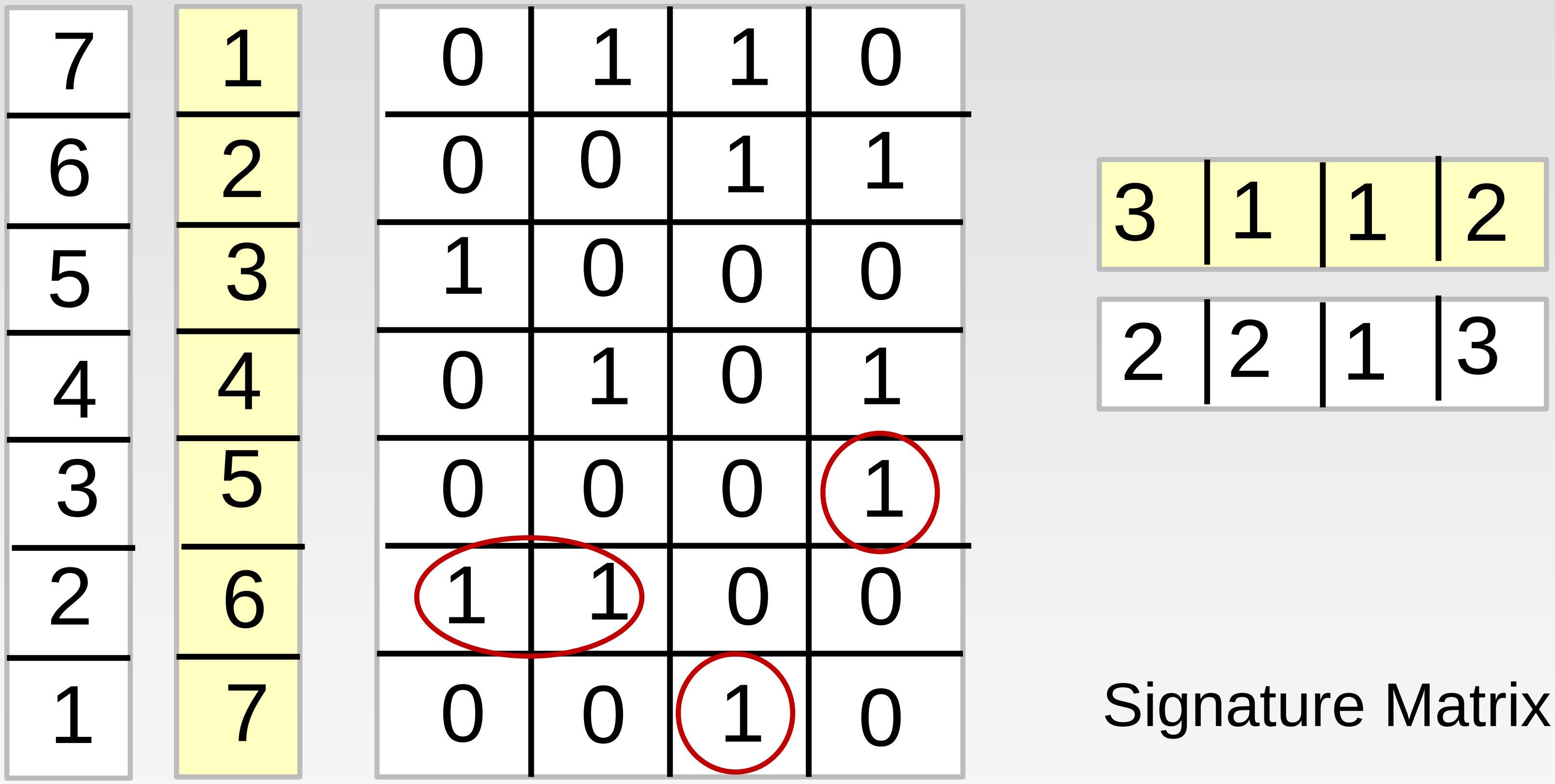
1	0	$\boxed{1}$	$\boxed{1}$	0
2	0	0	1	$\boxed{1}$
3	$\boxed{1}$	0	0	0
4	0	1	0	1
5	0	0	0	1
6	1	1	0	0
7	0	0	1	0

Input Matrix

3	1	1	2

Signature Matrix

Min-Hashing Example



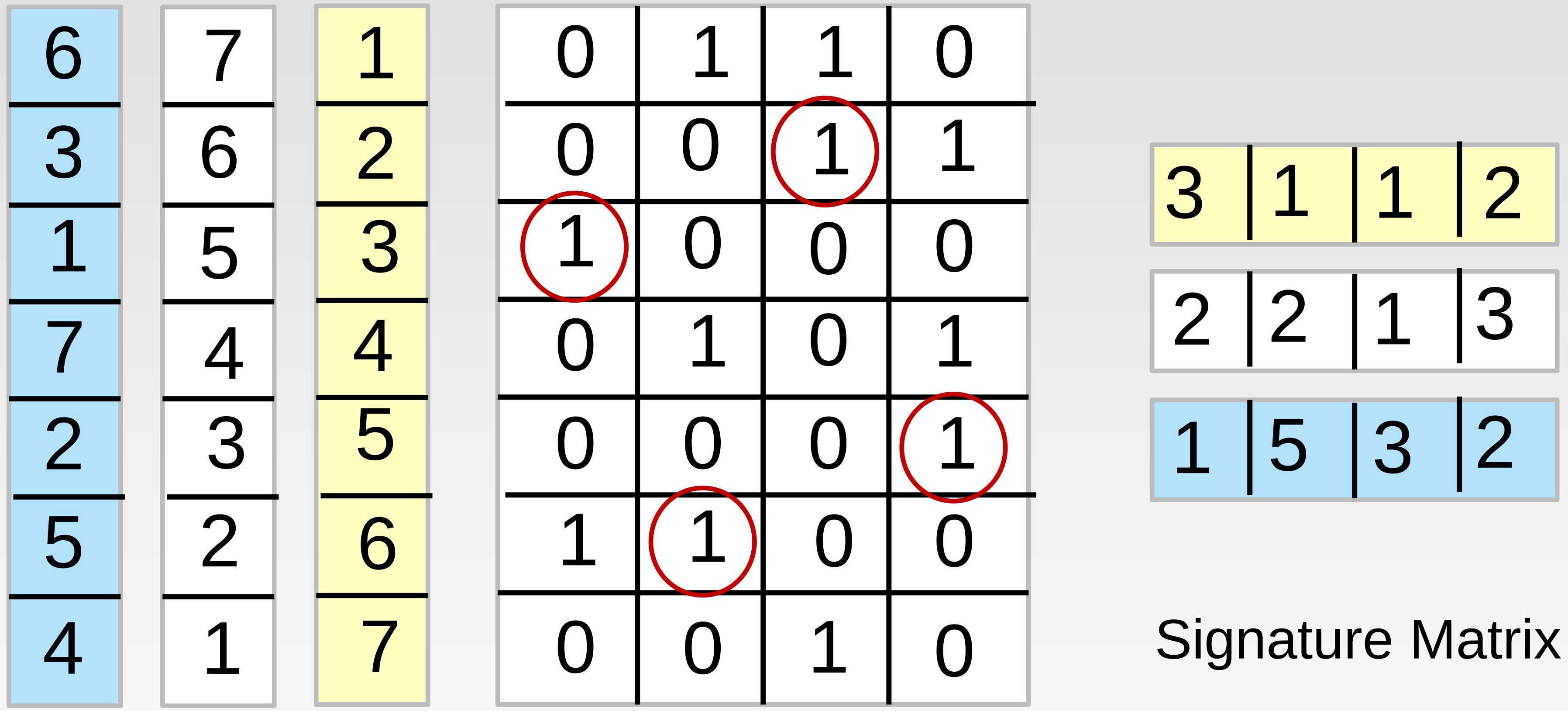
7	1	0	1	1	0
6	2	0	0	1	1
5	3	1	0	0	0
4	4	0	1	0	1
3	5	0	0	0	1
2	6	1	1	0	0
1	7	0	0	1	0

Input Matrix

3	1	1	2
2	2	1	3

Signature Matrix

Min-Hashing Example



6	7	1	0	1	1	0
3	6	2	0	0	1	1
1	5	3	1	0	0	0
7	4	4	0	1	0	1
2	3	5	0	0	0	1
5	2	6	1	1	0	0
4	1	7	0	0	1	0

Input Matrix

3	1	1	2
2	2	1	3
1	5	3	2

Signature Matrix

Min-Hashing Exar

Note: Another (equivalent) way is to store row indexes:

1	5	1	5
2	3	1	3
6	4	6	4

2nd element of the permutation is the first to map to a 1

Permutation π

2	4	3
3	2	4
7	1	7
6	3	2
1	6	6
5	7	1
4	5	5

Input matrix (Shingles x Documents)

1	0	1	0
1	0	0	1
0	1	0	1
0	1	0	1
0	1	0	1
1	0	1	0
1	0	1	0

Signature matrix *M*

2	1	2	1
2	1	4	1
1	2	1	2

4th element of the permutation is the first to map to a 1

The Min-Hash Property

0	0
0	0
1	**1**
0	0
0	1
1	0

- Choose a random permutation π
- Claim: $\Pr[h_\pi(C_1) = h_\pi(C_2)] = sim(C_1, C_2)$
- Why?
  - Let **X** be a doc (set of shingles), $\boldsymbol{y \in X}$ is a shingle
  - Then: $\Pr[\pi(y) = \min(\pi(X))] = 1/|X|$
    - It is equally likely that any $\boldsymbol{y \in X}$ is mapped to the ***min*** element
  - Let ***y*** be s.t. $\pi(y) = \min(\pi(C_1 \cup C_2))$
  - Then either: $\pi(y) = \min(\pi(C_1))$ if $y \in C_1$, **or**

    $\pi(y) = \min(\pi(C_2))$ if $y \in C_2$
  - So the prob. that **both** are true is the prob. $\boldsymbol{y} \in C_1 \cap C_2$
  - $\Pr[\min(\pi(C_1))=\min(\pi(C_2))]=|C_1 \cap C_2|/|C_1 \cup C_2|= sim(C_1, C_2)$

One of the two cols had to have 1 at position ***y***

Four Types of Rows

- **Given cols C_1 and C_2, rows may be classified as:**

	C_1	C_2
A	1	1
B	1	0
C	0	1
D	0	0

  - **a** = # rows of type A, etc.
- **Note:** **sim(C_1, C_2) = a/(a +b +c)**
- **Then:** **Pr**[$h(C_1) = h(C_2)$] = $Sim(C_1, C_2)$
  - Look down the cols C_1 and C_2 until we see a 1
  - If it's a type-*A* row, then $h(C_1) = h(C_2)$
    If a type-*B* or type-*C* row, then not

Similarity for Signatures

Permutation π

2	4	3
3	2	4
7	1	7
6	3	2
1	6	6
5	7	1
4	5	5

Input matrix (Shingles x Documents)

1	0	1	0
1	0	0	1
0	1	0	1
0	1	0	1
0	1	0	1
1	0	1	0
1	0	1	0

Signature matrix *M*

2	1	2	1
2	1	4	1
1	2	1	2

Similarities:

	1-3	2-4	1-2	3-4
Col/Col	0.75	0.75	0	0
Sig/Sig	0.67	1.00	0	0

Similarity for Signatures

- We know: **$\Pr[h_\pi(C_1) = h_\pi(C_2)] = sim(C_1, C_2)$**
- Now generalize to multiple hash functions

- **The *similarity of two signatures* is the fraction of the hash functions in which they agree**

- **Note:** Because of the Min-Hash property, the similarity of columns is the same as the expected similarity of their signatures

Min-Hash Signatures

- **Pick K=100 random permutations of the rows**
- Think of ***sig*(C)** as a column vector
- ***sig*(C)[i] =** according to the *i*-th permutation, the index of the first row that has a 1 in column *C*

$$\textit{sig}(C)[i] = \min(\pi_i(C))$$

- **Note:** The sketch (signature) of document *C* is small **~100 bytes!**

- **We achieved our goal! We "compressed" long bit vectors into short signatures**

Implementation Trick

- **Permuting rows even once is prohibitive**
- **Row hashing!**
  - Pick **K = 100** hash functions $\boldsymbol{k_i}$
  - Ordering under $\boldsymbol{k_i}$ gives a random row permutation!
- **One-pass implementation**
  - For each column ***C*** and hash-func. $\boldsymbol{k_i}$ keep a “slot” for the min-hash value
  - Initialize all ***sig(C)[i]*** $= \infty$
  - **Scan rows looking for 1s**
    - Suppose row ***j*** has 1 in column ***C***
    - Then for each $\boldsymbol{k_i}$:
      - If $\boldsymbol{k_i(j) < sig(C)[i]}$, then $\boldsymbol{sig(C)[i] \leftarrow k_i(j)}$

How to pick a random hash function h(x)?
Universal hashing:
$h_{a,b}(x)=((a \cdot x+b) \bmod p) \bmod N$
where:
a,b … random integers
p … prime number (p > N)

Implementation Example

Row	S_1	S_2	S_3	S_4	$x+1 \mod 5$	$3x+1 \mod 5$
0	1	0	0	1	1	1
1	0	0	1	0	2	4
2	0	1	0	1	3	2
3	1	0	1	1	4	0
4	0	0	1	0	0	3

- 0. Initialize all ***sig(C)[i]* = ∞**

	S_1	S_2	S_3	S_4
h_1	∞	∞	∞	∞
h_2	∞	∞	∞	∞

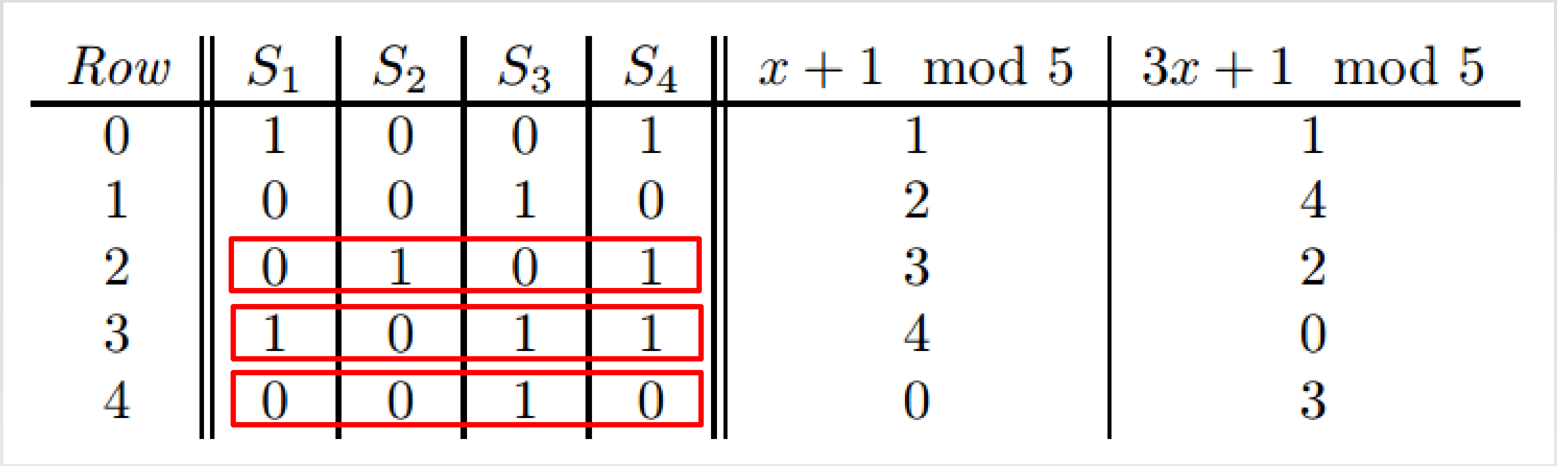
- Row 0: we see that the values of $h_1(0)$ and $h_2(0)$ are both 1, thus $sig(S_1)[0] = 1$, $sig(S_1)[1] = 1$, $sig(S_4)[0] = 1$, $sig(S_4)[1] = 1$,

	S_1	S_2	S_3	S_4
h_1	1	∞	∞	1
h_2	1	∞	∞	1

- Row 1, we see $h_1(1) = 2$ and $h_2(1) = 4$, thus $sig(S_3)[0] = 2$, $sig(S_3)[1] = 4$

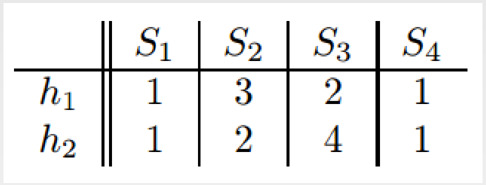
	S_1	S_2	S_3	S_4
h_1	1	∞	2	1
h_2	1	∞	4	1

Implementation Example



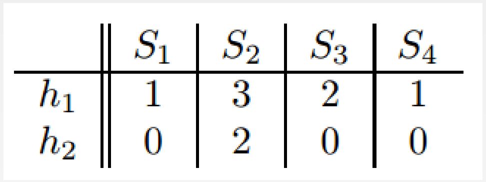
Row	S_1	S_2	S_3	S_4	$x+1 \mod 5$	$3x+1 \mod 5$
0	1	0	0	1	1	1
1	0	0	1	0	2	4
2	0	1	0	1	3	2
3	1	0	1	1	4	0
4	0	0	1	0	0	3

- Row 2: $h_1(2) = 3$ and $h_2(2) = 2$, thus
  $sig(S_2)[0] = 3$, $sig(S_2)[1] = 2$, no update for S_4



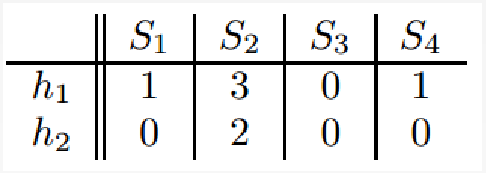
	S_1	S_2	S_3	S_4
h_1	1	3	2	1
h_2	1	2	4	1

- Row 3: $h_1(3) = 4$ and $h_2(3) = 0$, update
  $sig(S_1)[1] = 0$, $sig(S_3)[1] = 0$, $sig(S_4)[1] = 0$,



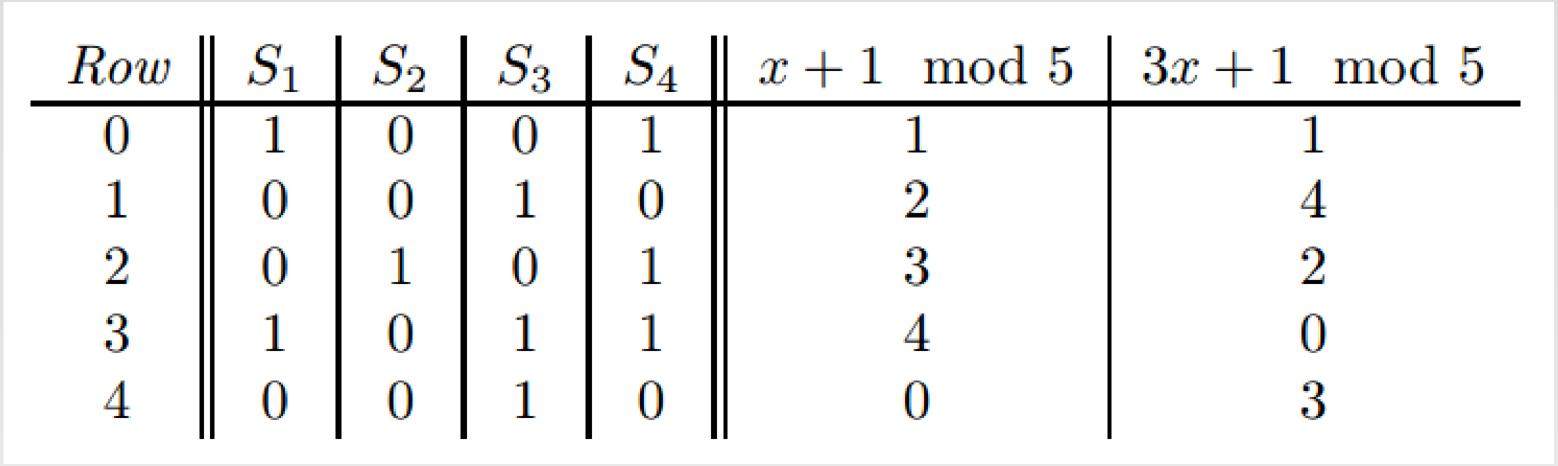
	S_1	S_2	S_3	S_4
h_1	1	3	2	1
h_2	0	2	0	0

- Row 4: $h_1(4) = 0$ and $h_2(4) = 3$, update
  $sig(S_3)[0] = 0$,

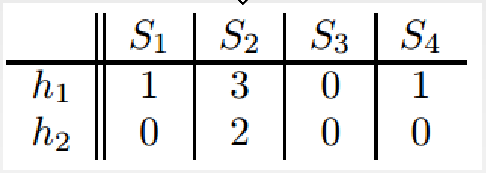


	S_1	S_2	S_3	S_4
h_1	1	3	0	1
h_2	0	2	0	0

Implementation Example



Row	S_1	S_2	S_3	S_4	$x+1 \mod 5$	$3x+1 \mod 5$
0	1	0	0	1	1	1
1	0	0	1	0	2	4
2	0	1	0	1	3	2
3	1	0	1	1	4	0
4	0	0	1	0	0	3



	S_1	S_2	S_3	S_4
h_1	1	3	0	1
h_2	0	2	0	0

- We can estimate the Jaccard similarities of the underlying sets from this signature matrix.
  - Signature matrix: SIM(S_1, S_4) = 1.0
  - Jaccard Similarity: SIM(S_1, S_4) = 2/3

References

- Chapter 3 of Mining of Massive Datasets.

End of Chapter 7.1