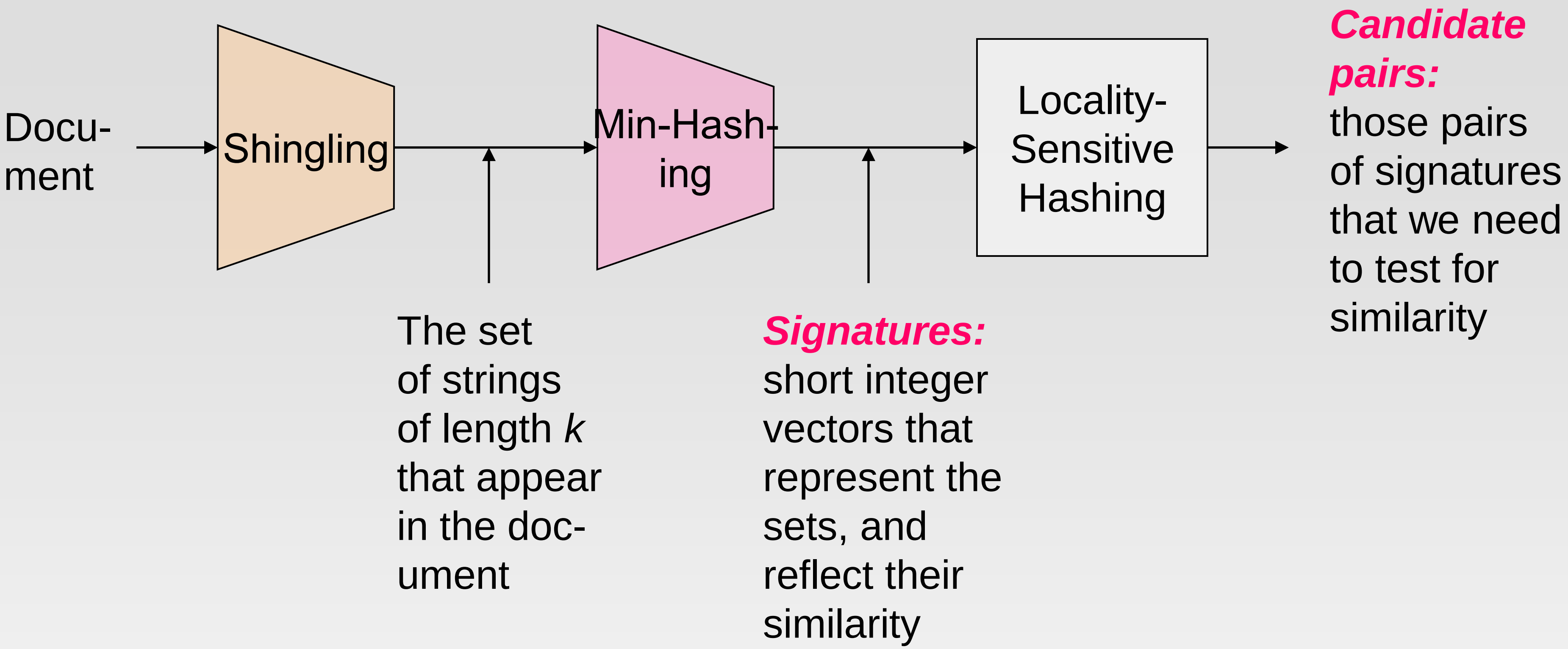
COMP9313: Big Data Management

Lecturer: Xin Cao

Course web site: http://www.cse.unsw.edu.au/~cs9313/

Chapter 7.2: Finding Similar Items



Step 3: ***Locality-Sensitive Hashing:***
Focus on pairs of signatures likely to be from similar documents

LSH: First Cut

2	1	4	1
1	2	1	2
2	1	2	1

- **Goal:** Find documents with Jaccard similarity at least ***s*** (for some similarity threshold, e.g., ***s***=0.8)

- **LSH – General idea:** Use a function ***f(x,y)*** that tells whether ***x*** and ***y*** is a ***candidate pair***: a pair of elements whose similarity must be evaluated

- **For Min-Hash matrices:**
  - Hash columns of signature matrix ***M*** to many buckets
  - Each pair of documents that hashes into the same bucket is a **candidate pair**

Candidates from Min-Hash

2	1	4	1
1	2	1	2
2	1	2	1

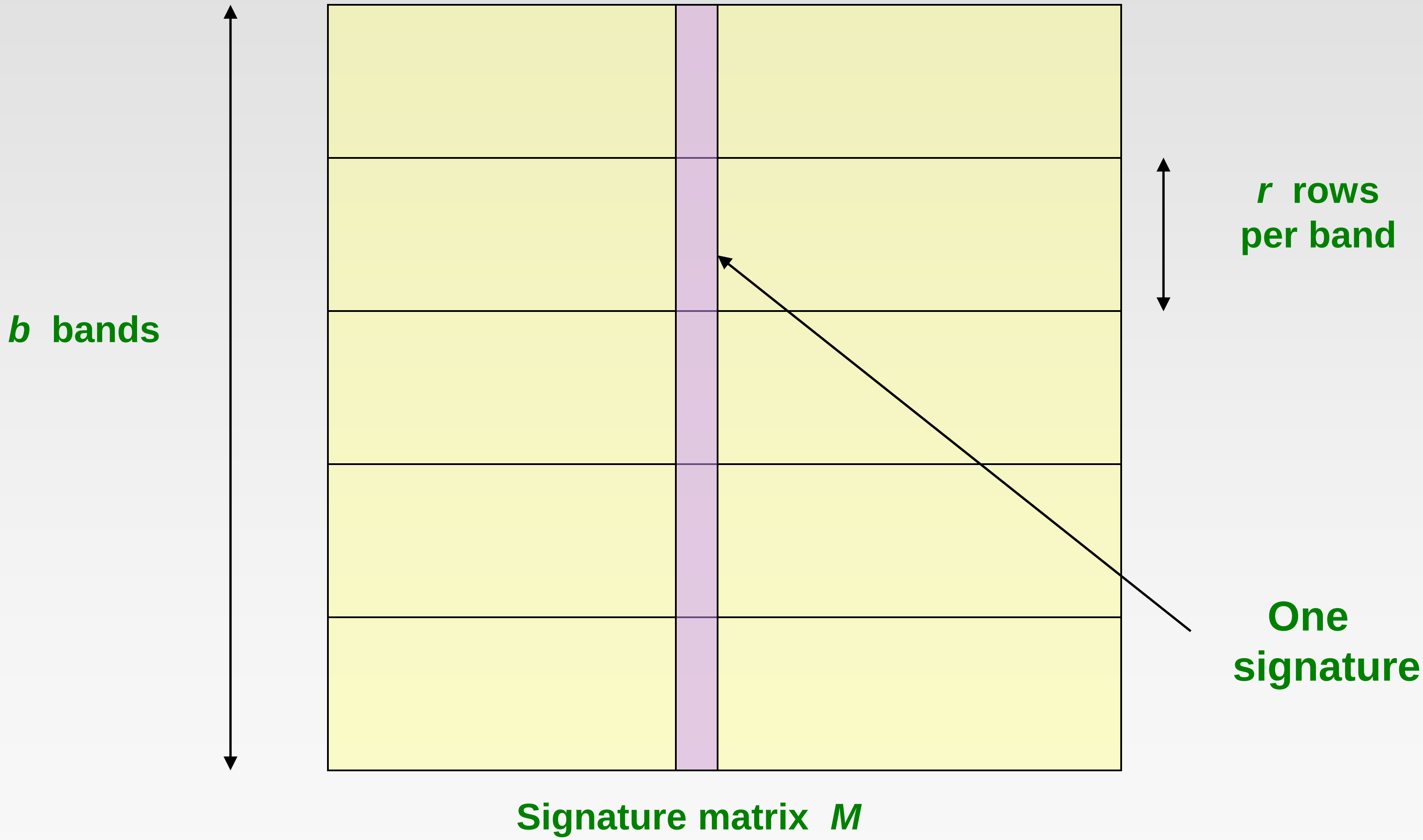
- **Pick a similarity threshold *s* (0 < s < 1)**
- Columns ***x*** and ***y*** of ***M*** are a **candidate pair** if their signatures agree on at least fraction ***s*** of their rows:
  M* (*i*, *x*) = *M* (*i*, *y*)** for at least frac. ***s values of ***i***
  - We expect documents ***x*** and ***y*** to have the same (Jaccard) similarity as their signatures

LSH for Min-Hash

2	1	4	1
1	2	1	2
2	1	2	1

- **Big idea: Hash columns of signature matrix *M* several times**
- Arrange that (only) **similar columns** are likely to **hash to the same bucket**, with high probability
- **Candidate pairs are those that hash to the same bucket**

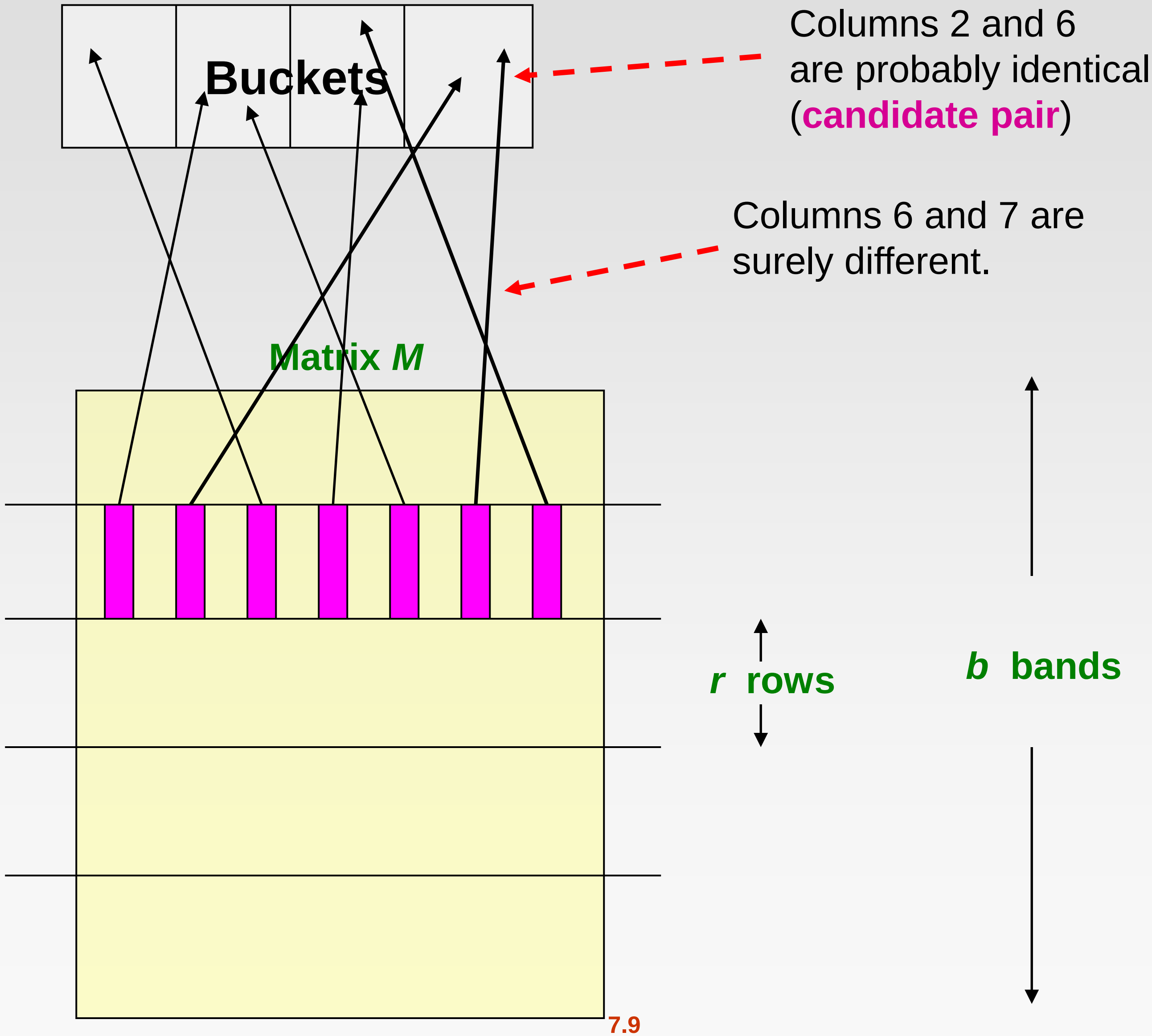
Partition *M* into *b* Bands



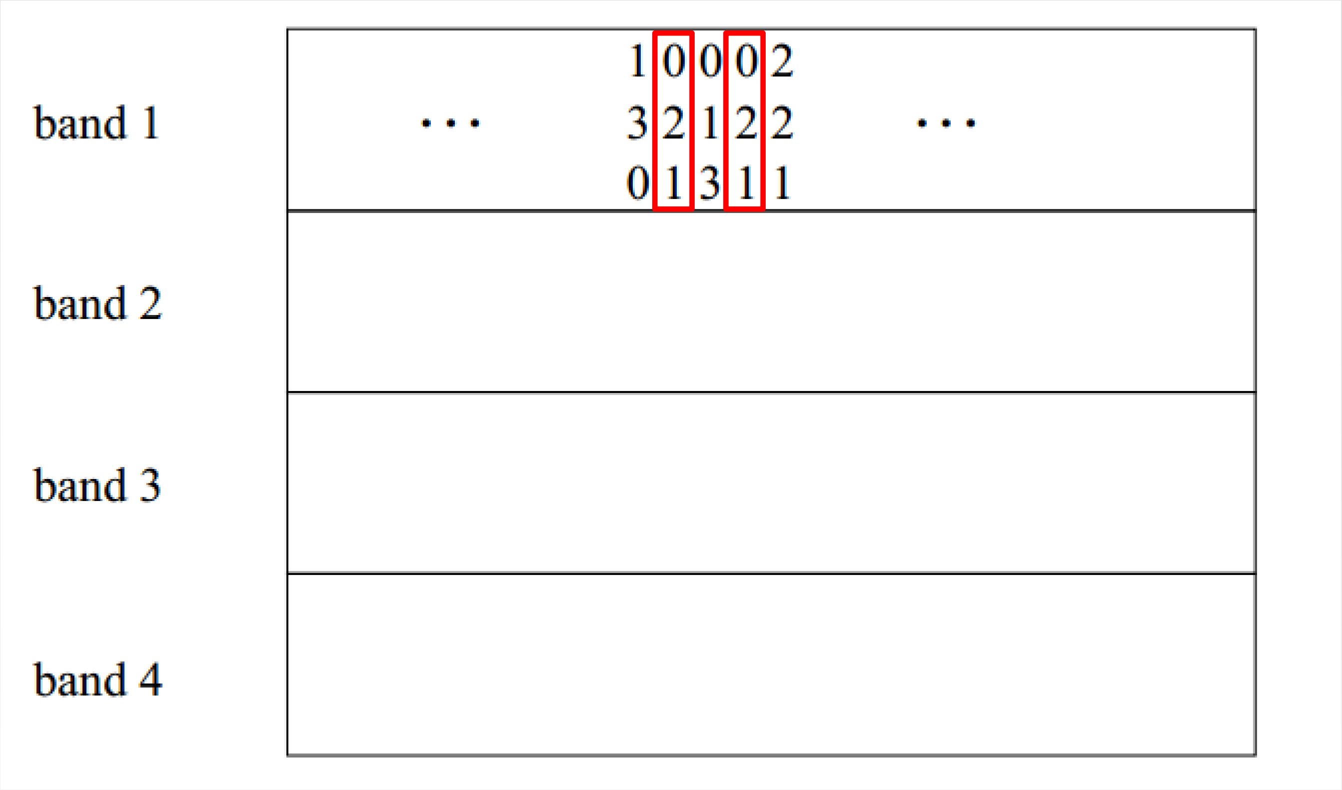
Partition M into Bands

- Divide matrix ***M*** into ***b*** bands of ***r*** rows
- For each band, hash its portion of each column to a hash table with ***k*** buckets
  - Make ***k*** as large as possible
- ***Candidate*** column pairs are those that hash to the same bucket for ≥ **1** band
- Tune ***b*** and ***r*** to catch most similar pairs, but few non-similar pairs

Hashing Bands



Hashing Bands



- Regardless of what those columns look like in the other three bands, this pair of columns will be a candidate pair
- Two columns that do not agree in band 1 have three other chances to become a candidate pair; they might be identical in any one of these other bands.

Simplifying Assumption

- There are **enough buckets** that columns are unlikely to hash to the same bucket unless they are **identical** in a particular band

- Hereafter, we assume that "**same bucket**" means "**identical in that band**"

- Assumption needed only to simplify analysis, not for correctness of algorithm

Example of Bands

Assume the following case:

- Suppose 100,000 columns of ***M*** (100k docs)
- Signatures of 100 integers (rows)
- Therefore, signatures take 40Mb
- Choose ***b*** = 20 bands of ***r*** = 5 integers/band

- **Goal:** Find pairs of documents that are at least ***s*** = *0.8* similar

C_1, C_2 are 80% Similar

- **Find pairs of ≥ *s*=0.8** similarity, set **b**=20, **r**=5
- **Assume:** sim(C_1, C_2) = 0.8
  - Since sim(C_1, C_2) ≥ **s**, we want C_1, C_2 to be a **candidate pair**: We want them to hash to at **least 1 common bucket** (at least one band is identical)
- **Probability C_1, C_2 identical in one particular band:** $(0.8)^5 = 0.328$
- Probability C_1, C_2 are ***not*** similar in all of the 20 bands: $(1-0.328)^{20} = 0.00035$
  - i.e., about 1/3000th of the 80%-similar column pairs are **false negatives** (we miss them)
  - **We would find 99.965% pairs of truly similar documents**

C_1, C_2 are 30% Similar

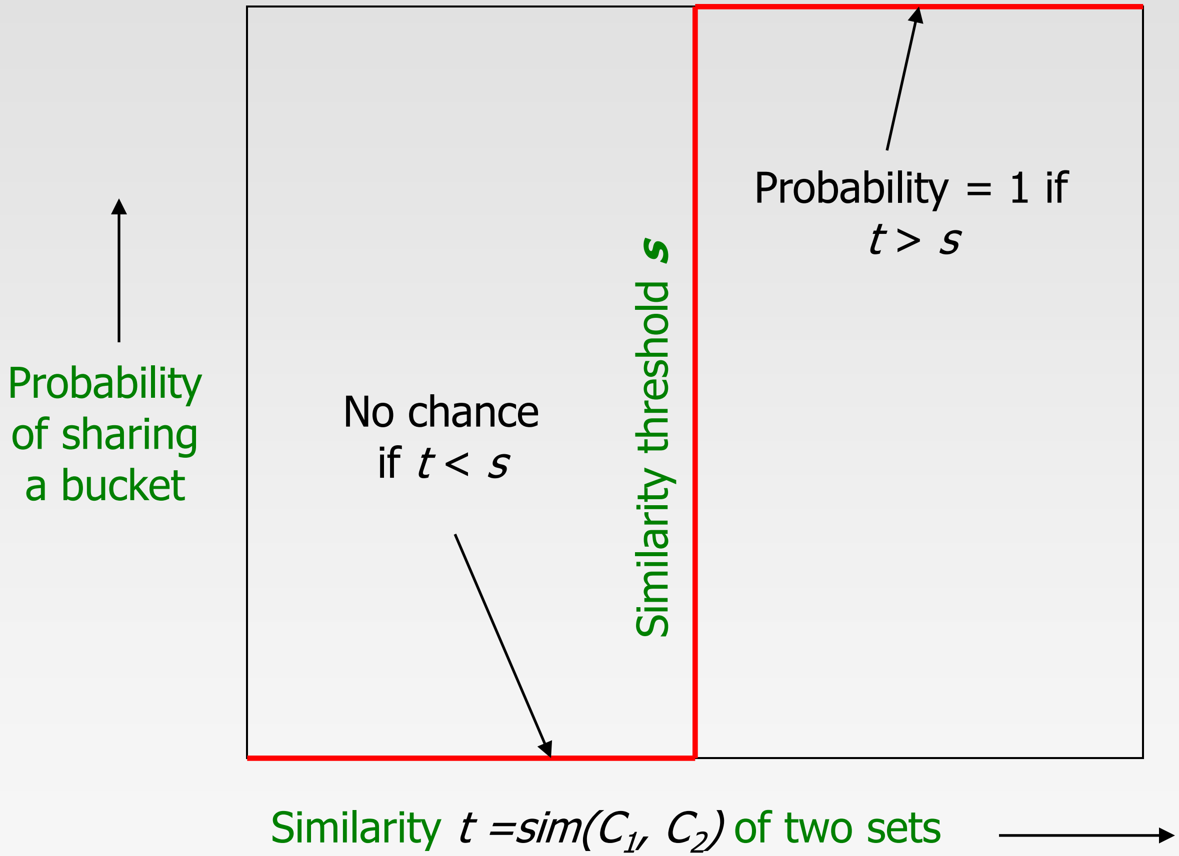
- **Find pairs of ≥ *s*=0.8 similarity, set b=20, r=5**
- **Assume:** sim(C_1, C_2) = 0.3
  - Since sim(C_1, C_2) < **s** we want C_1, C_2 to hash to **NO common buckets** (all bands should be different)
- **Probability C_1, C_2 identical in one particular band:** $(0.3)^5 = 0.00243$
- Probability C_1, C_2 identical in at least 1 of 20 bands: $1 - (1 - 0.00243)^{20} = 0.0474$
  - In other words, approximately 4.74% pairs of docs with similarity 0.3% end up becoming **candidate pairs**
    - They are **false positives** since we will have to examine them (they are candidate pairs) but then it will turn out their similarity is below threshold **s**

LSH Involves a Tradeoff

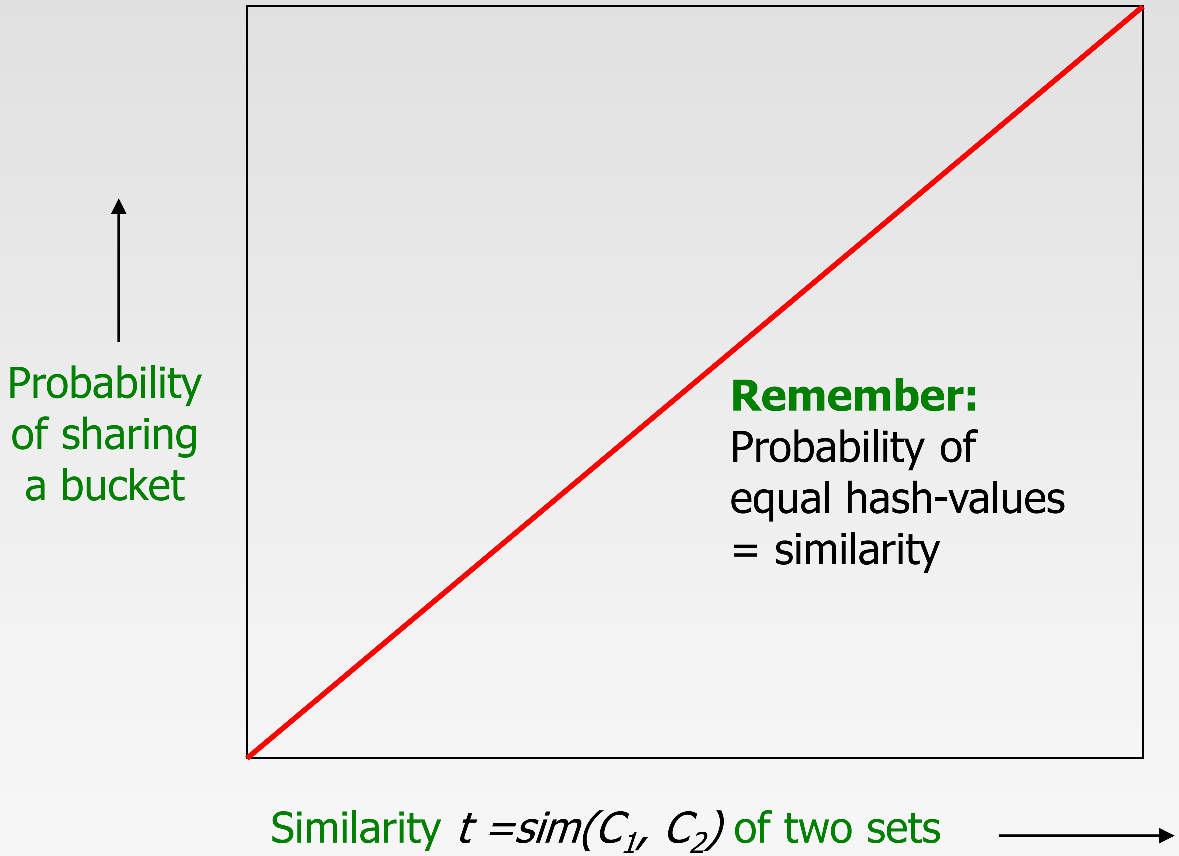
- **Pick:**
  - The number of Min-Hashes (rows of ***M***)
  - The number of bands ***b***, and
  - The number of rows ***r*** per band to balance false positives/negatives

- **Example:** If we had only 15 bands of 5 rows, the number of false positives would go down, but the number of false negatives would go up

Analysis of LSH – What We Want



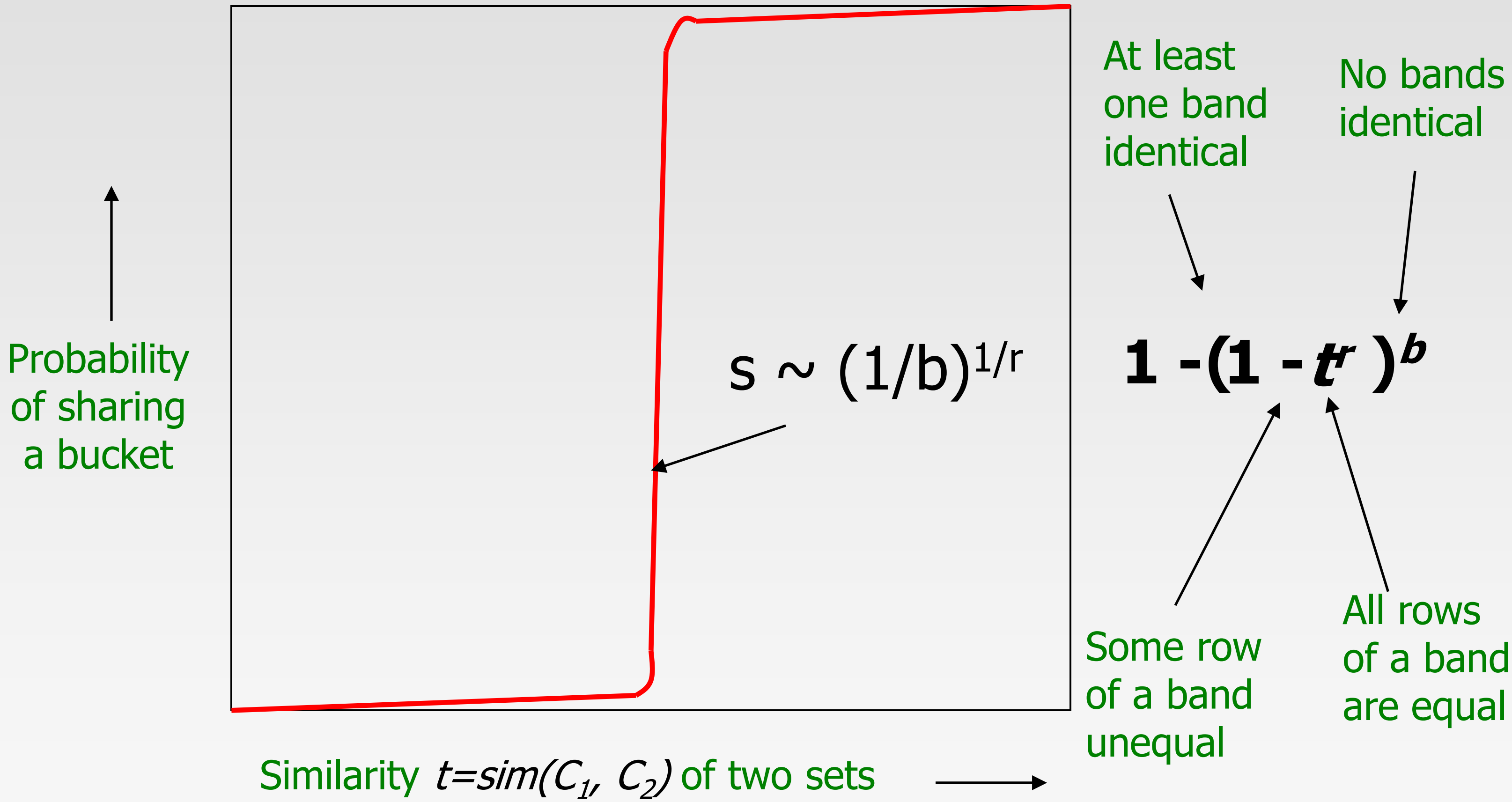
What 1 Band of 1 Row Gives You



b **bands,** ***r*** **rows/band**

- The probability that the minhash signatures for the documents agree in any one particular row of the signature matrix is t ($sim(C_1, C_2)$)
- Pick any band (r rows)
  - Prob. that all rows in band equal = t^r
  - Prob. that some row in band unequal = $1 - t^r$
- Prob. that no band identical = $(1 - t^r)^b$
- Prob. that at least 1 band identical = $1 - (1 - t^r)^b$

What *b* Bands of *r* Rows Gives You



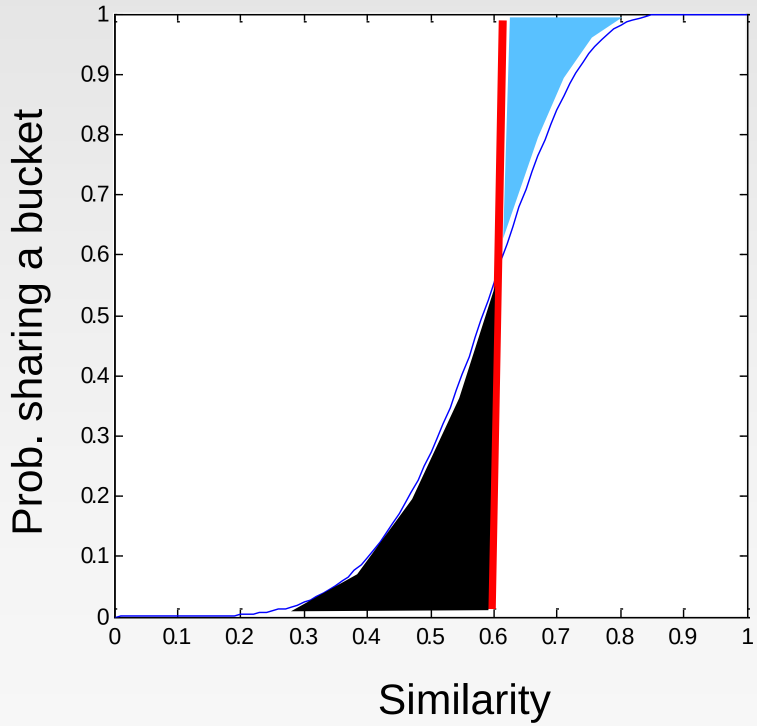
Example: $b = 20$, $r = 5$

- Similarity threshold s
- Prob. that at least 1 band is identical:

s	$1-(1-s^r)^b$
.2	.006
.3	.047
.4	.186
.5	.470
.6	.802
.7	.975
.8	.9996

Picking *r* and *b*: The S-curve

- **Picking *r* and *b* to get the best S-curve**
  - 50 hash-functions (r=5, b=10)



Blue area: False Negative rate
Black area: False Positive rate

LSH Summary

- Tune ***M, b, r*** to get almost all pairs with similar signatures, but eliminate most pairs that do not have similar signatures
- Check in main memory that **candidate pairs** really do have **similar signatures**
- **Optional:** In another pass through data, check that the remaining candidate pairs really represent similar documents

Summary: 3 Steps

- **Shingling:** Convert documents to sets
  - We used hashing to assign each shingle an ID
- **Min-Hashing:** Convert large sets to short signatures, while preserving similarity
  - We used **similarity preserving hashing** to generate signatures with property $\mathbf{Pr}[h_\pi(C_1) = h_\pi(C_2)] = sim(C_1, C_2)$
  - We used hashing to get around generating random permutations
- **Locality-Sensitive Hashing:** Focus on pairs of signatures likely to be from similar documents
  - We used hashing to find **candidate pairs** of similarity $\geq$ **s**

Distance Measures

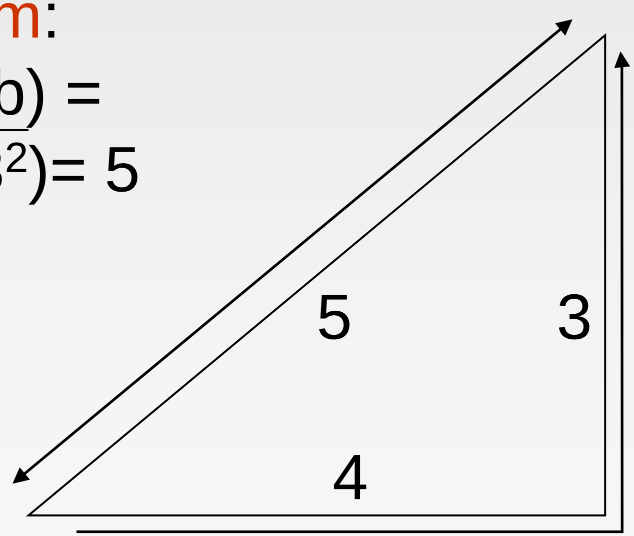
- Generalized LSH is based on some kind of "distance" between points.
  - Similar points are "close."
- Example: Jaccard similarity is not a distance; 1 minus Jaccard similarity is.
- *d* is a *distance measure* if it is a function from pairs of points to real numbers such that:
  1. $d(x,y) \geq 0$.
  2. $d(x,y) = 0$ iff $x = y$.
  3. $d(x,y) = d(y,x)$.
  4. $d(x,y) \leq d(x,z) + d(z,y)$ (*triangle inequality*).

Some Euclidean Distances

- *L_2 norm*: d(x,y) = square root of the sum of the squares of the differences between *x* and *y* in each dimension.
  - The most common notion of "distance."
- *L_1 norm*: sum of the differences in each dimension.
  - *Manhattan distance* = distance if you had to travel along coordinates only.

b = (9,8)

L_2-norm:
dist(a,b) = $\sqrt{(4^2+3^2)} = 5$



a = (5,5)

L_1-norm:
dist(a,b) =4+3 = 7

Some Non-Euclidean Distances

- *Jaccard distance* for sets = 1 minus Jaccard similarity.
- *Cosine distance* for vectors = angle between the vectors.
- *Edit distance* for strings = number of inserts and deletes to change one string into another.

Cosine Distance

- Think of a point as a vector from the origin [0,0,…,0] to its location.
- Two points' vectors make an angle, whose cosine is the normalized dot-product of the vectors: $p_1.p_2/|p_2||p_1|$.
  - Example: p_1 = [1,0,2,-2,0]; p_2 = [0,0,3,0,0].
  - $p_1.p_2 = 6$; $|p_1| = |p_2| = \sqrt{9} = 3$.
  - $\cos(\theta) = 6/9$; θ is about 48 degrees.

Edit Distance

- The *edit distance* of two strings is the number of inserts and deletes of characters needed to turn one into the other.
- An equivalent definition: d(x,y) = |x| + |y| - 2|LCS(x,y)|.
  - LCS = *longest common subsequence* = any longest string obtained both by deleting from *x* and deleting from *y*.
- Example:
  - *x* = *abcde* ; *y* = *bcduve*.
  - Turn *x* into *y* by deleting *a*, then inserting *u* and *v* after *d*.
    - Edit distance = 3.
  - Or, computing edit distance through the LCS, note that LCS(x,y) = *bcde*.
  - Then:|x| + |y| - 2|LCS(x,y)| = 5 + 6 –2*4 = 3 = edit distance.

Hash Functions Decide Equality

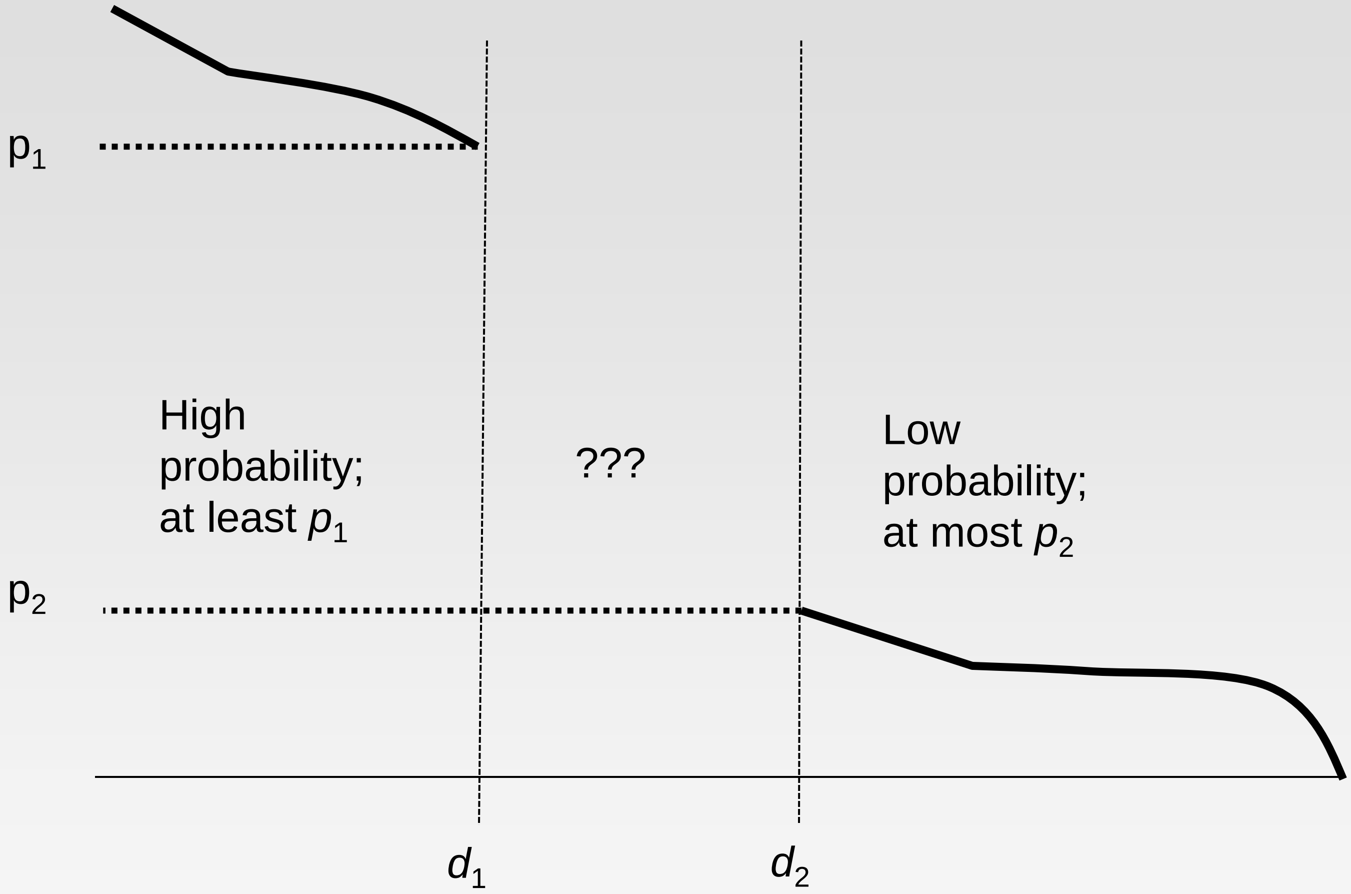
- There is a subtlety about what a "hash function" is, in the context of LSH families.
- A hash function h really takes two elements x and y, and returns a decision whether x and y are candidates for comparison.
- Example: the family of minhash functions computes minhash values and says "yes" iff they are the same.
- Shorthand: "h(x) = h(y)" means h says "yes" for pair of elements x and y.

LSH Families Defined

- Suppose we have a space *S* of points with a distance measure *d*.
- A family **H** of hash functions is said to be *(d_1,d_2,p_1,p_2)-sensitive* if for any *x* and *y* in *S*:

1. If $d(x,y) \leq d_1$, then the probability over all *h* in **H**, that h(x) = h(y) is at least p_1.
2. If $d(x,y) \geq d_2$, then the probability over all *h* in **H**, that h(x) = h(y) is at most p_2.

LS Families: Illustration



Example: LS Family – (2)

- **Claim**: **H** is a (1/3, 3/4, 2/3, 1/4)-sensitive family for *S* and *d*.

If distance $\geq$ 3/4
(so similarity $\leq$ 1/4)

Then probability
that minhash values
agree is $\leq$ 1/4

If distance $\leq$ 1/3
(so similarity $\geq$ 2/3)

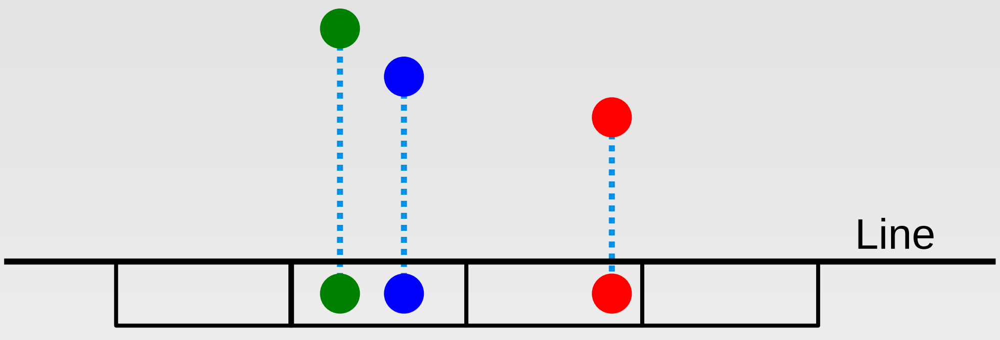
Then probability
that minhash values
agree is $\geq$ 2/3

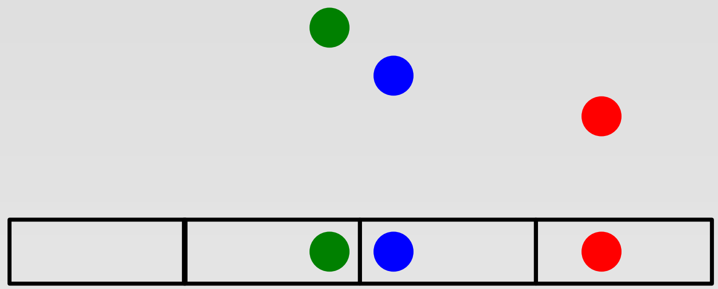
For Jaccard similarity, minhashing gives us a
$(d_1,d_2,(1-d_1),(1-d_2))$-sensitive family for any $d_1 < d_2$.

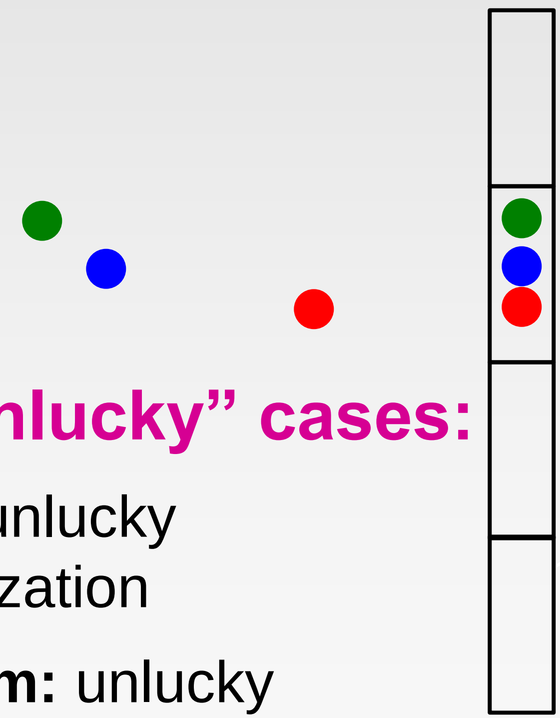
LSH for Euclidean Distance

- **Idea: Hash functions correspond to lines**
- Partition the line into buckets of size ***a***
- **Hash each point to the bucket containing its projection onto the line**
  - An element of the "Signature" is a bucket id for that given projection line
- **Nearby points are always close**; distant points are rarely in same bucket

Projection of Points

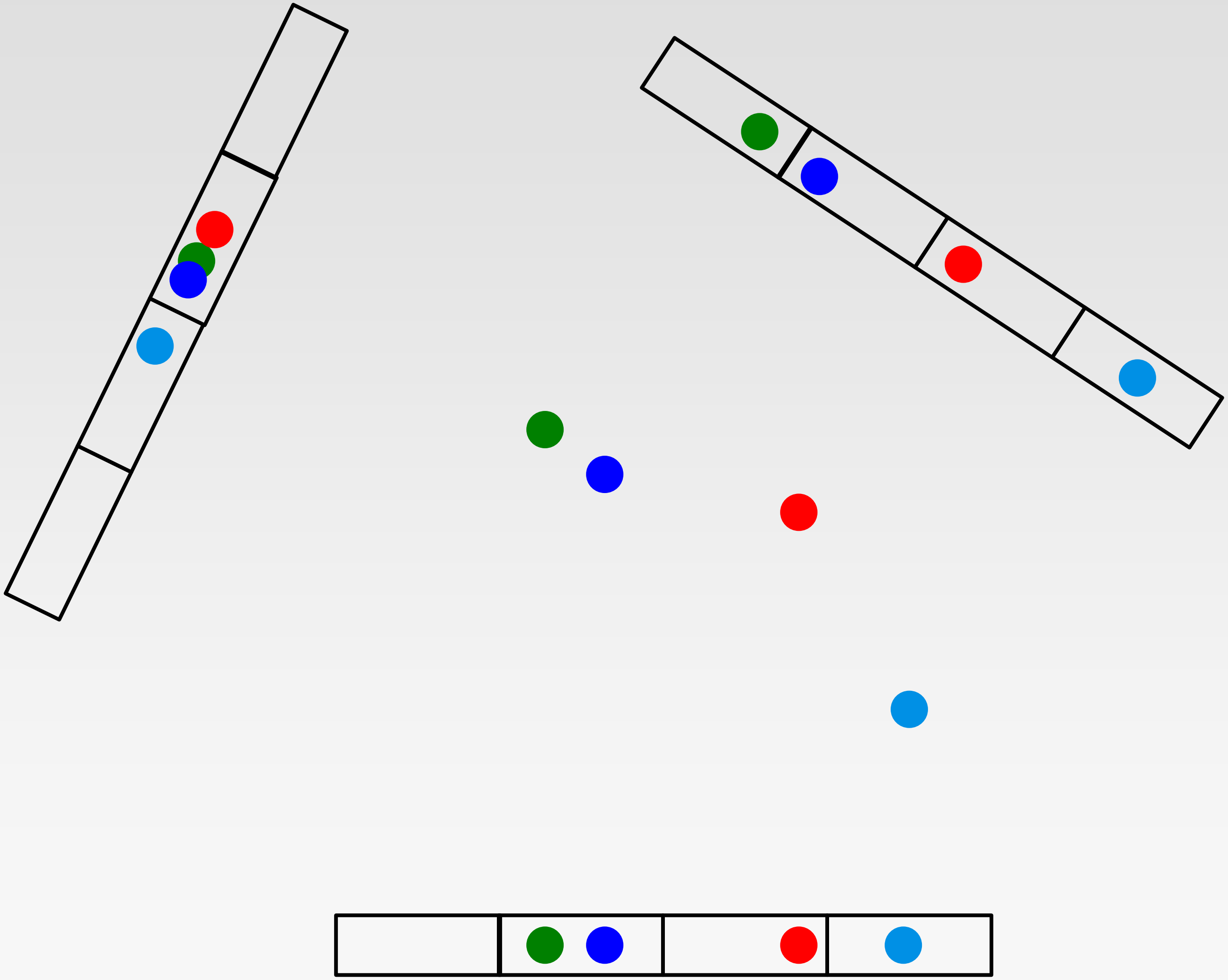




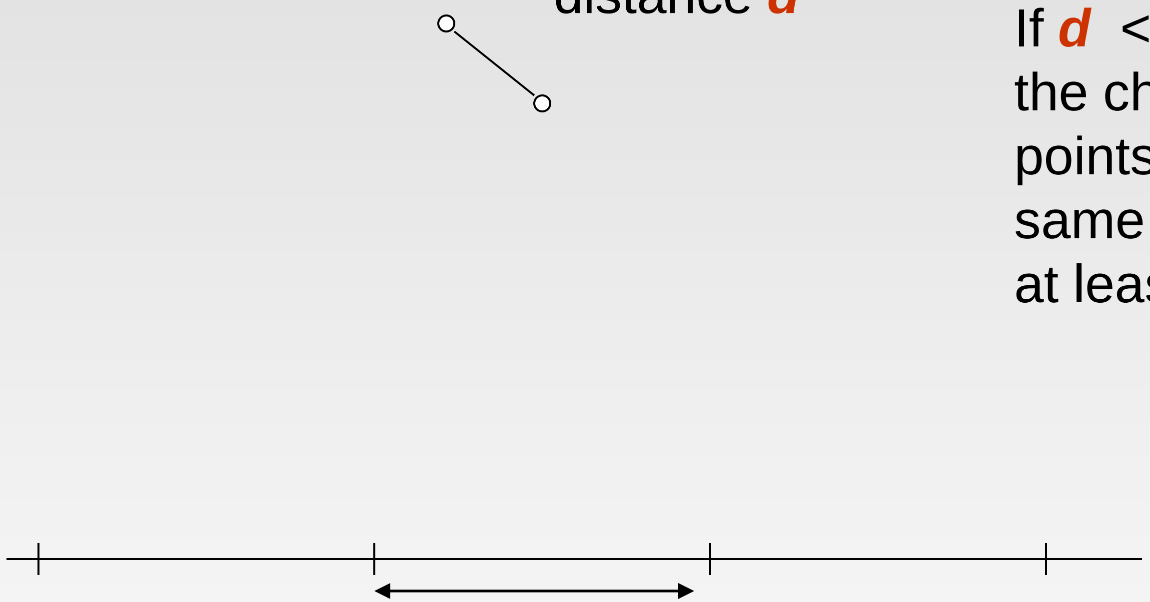


- **"Lucky" case:**
  - Points that are close hash in the same bucket
  - Distant points end up in different buckets
- **Two "unlucky" cases:**
  - **Top:** unlucky quantization
  - **Bottom:** unlucky projection

Multiple Projections



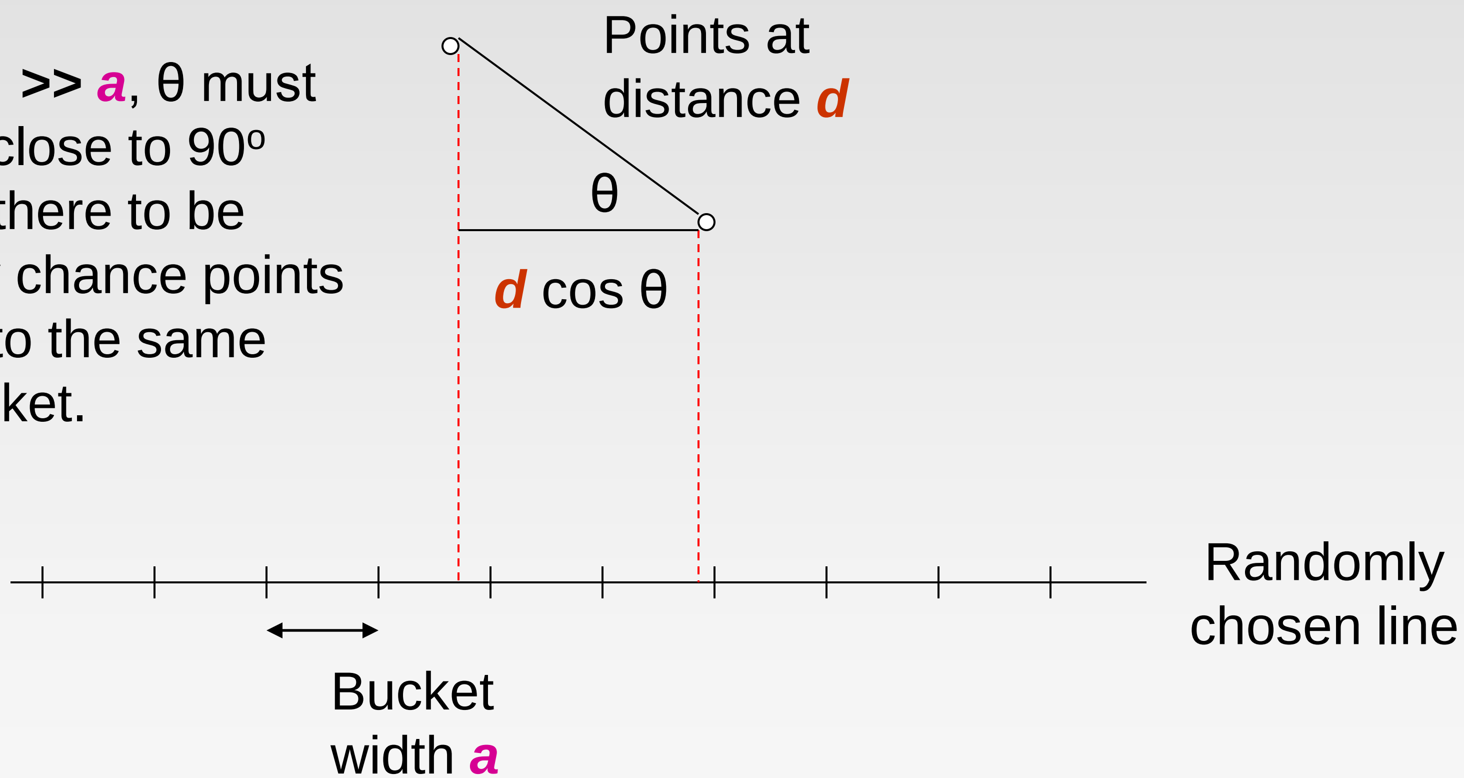
Projection of Points



If *d* << *a*, then the chance the points are in the same bucket is at least **1 – *d*/*a***.

Projection of Points

If ***d*** **>>** ***a***, θ must be close to 90º for there to be any chance points go to the same bucket.

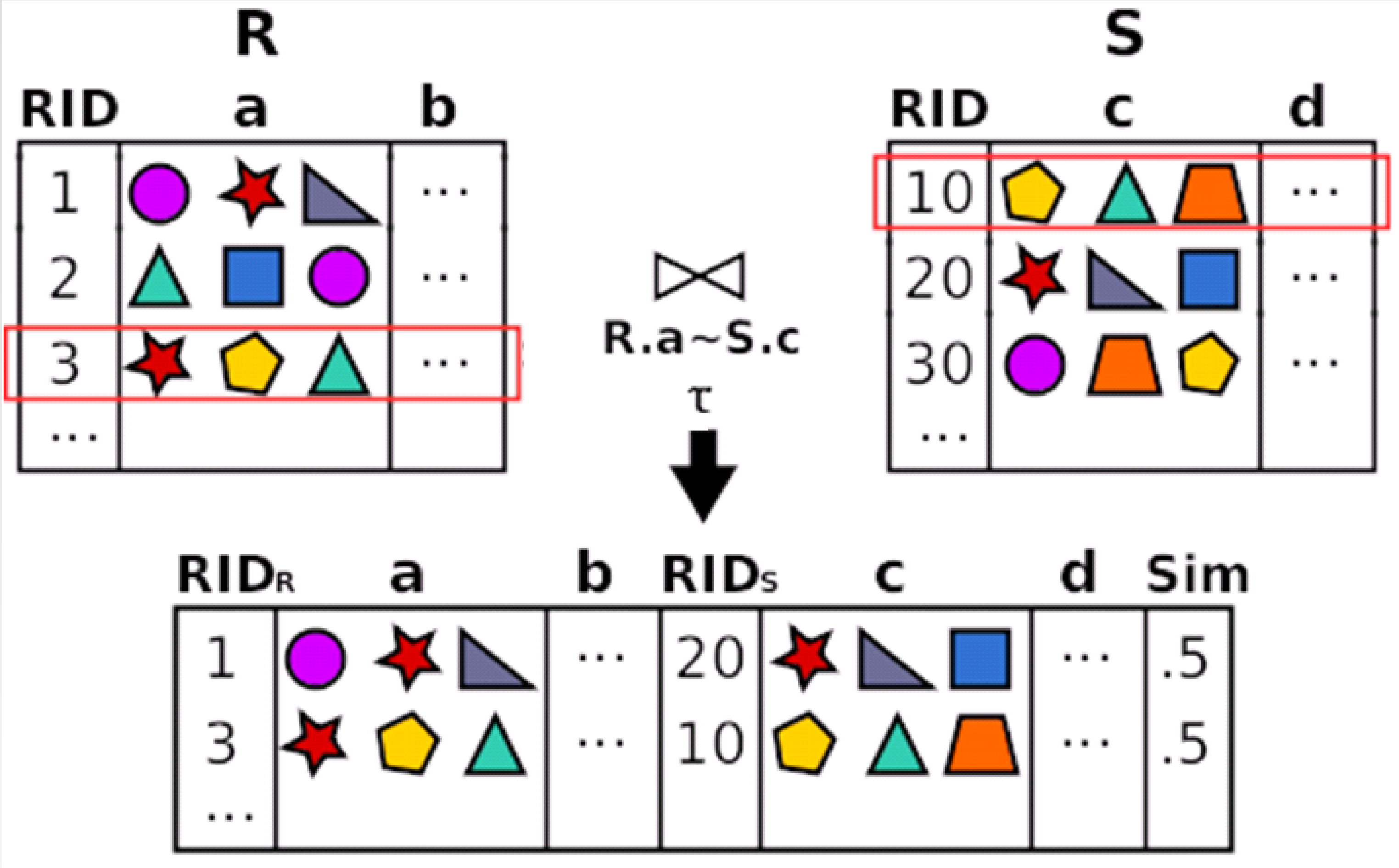


An LS-Family for Euclidean Distance

- If points are distance $d \leq a/2$, prob, they are in same bucket $\geq 1 - d/a = ½$
- If points are distance $d \geq 2a$ apart, then they can be in the same bucket only if $d \cos \theta \leq a$
  - $\cos \theta \leq ½$
  - $60 \leq \theta \leq 90$, i.e., at most 1/3 probability
- Yields a ***(a/2, 2a, 1/2, 1/3)**-sensitive* family of hash functions for any ***a***

Part 2: Exact Approach to Finding Similar Records

Set-Similarity Join



Finding pairs of records with a **similarity** on their join attributes > t

Set-Similarity Join

- Given two collections of records R and S, a similarity function sim(., .), and a threshold τ, the set similarity join between R and S, is to find all record pairs r (from R) and s (from S), such that sim(r, s) >= τ.

id	set
r_1	$\{e_1, e_4, e_5, e_6\}$
r_2	$\{e_2, e_3, e_6\}$
r_3	$\{e_4, e_5, e_6\}$

(a) $\mathcal{R}$ sets

id	set
s_1	$\{e_1, e_4, e_6\}$
s_2	$\{e_2, e_5, e_6\}$
s_3	$\{e_3, e_5\}$

(b) $\mathcal{S}$ sets

- Given the above example, and set τ=0.5, the results are： (r1, s1) (similarity 0.75), (r2, s2) (similarity 0.5), (r3, s1) (similarity 0.5), (r3, s2) (similarity 0.5).
- LSH can solve this problem approximately.

Application: Record linkage

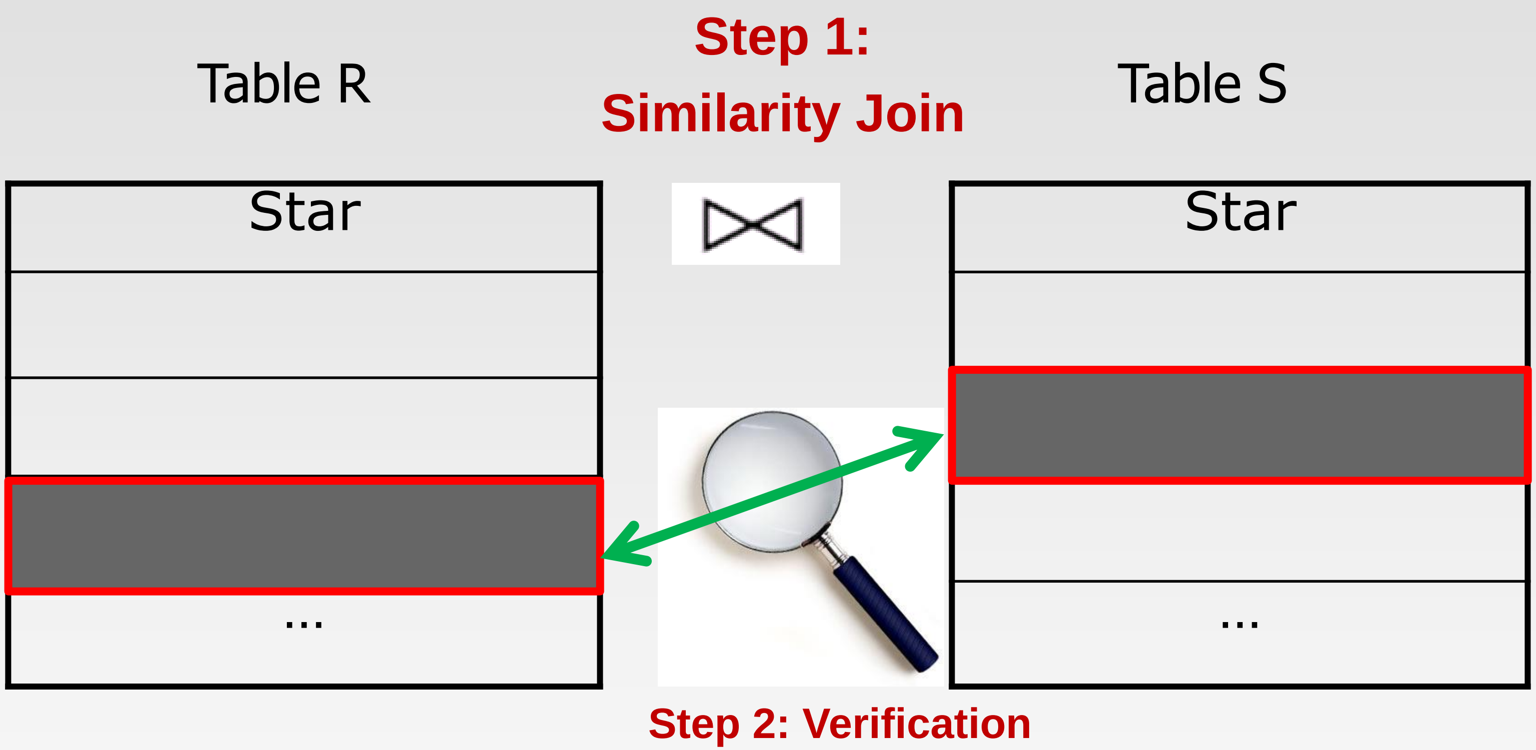
Table R

Star
Keanu Reeves
Samuel Jackson
Schwarzenegger
…

Table S

Star
Keanu Reeves
Samuel L. Jackson
Schwarzenegger
…

Two-step Solution



Self-Join

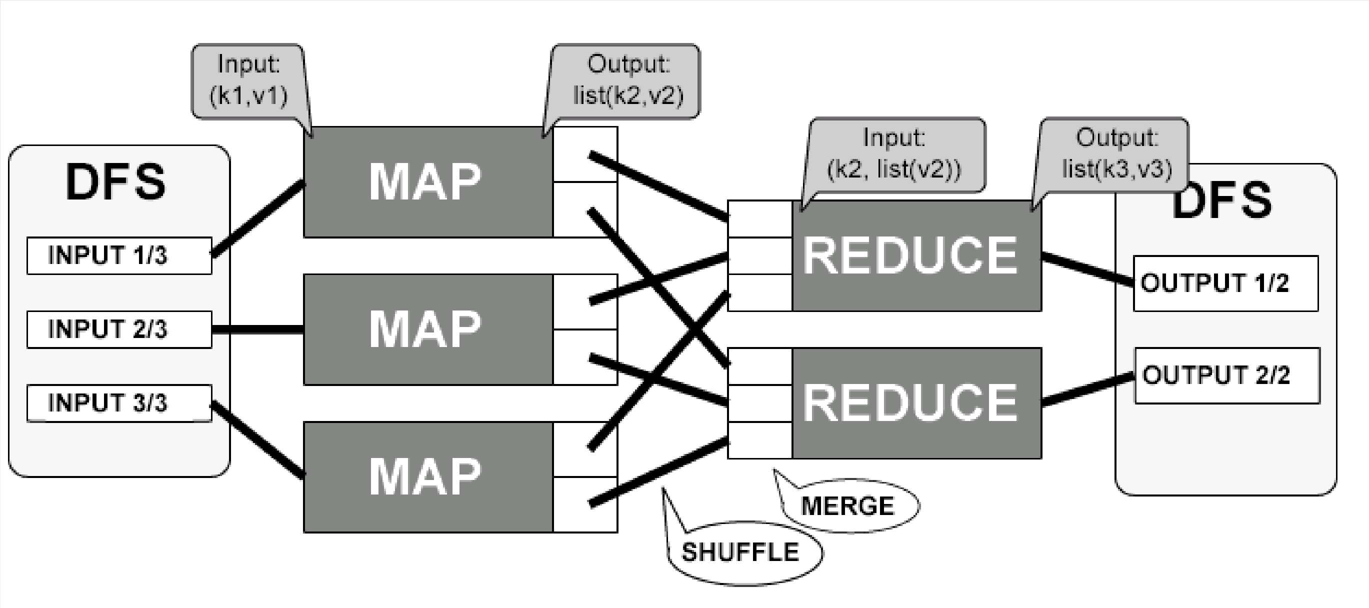
- Given a collection of records R, a similarity function **sim**(., .), and a threshold τ, the set similarity self-join on R, is to find all record pairs r and s from R, such that **sim**(r, s) >= τ.

id	record
0	1 4 5 6
1	2 3 6
2	4 5 6
3	1 4 6
4	2 5 6
5	3 5

pair	similarity
(0,2)	0.75
(0,3)	0.75
(1,4)	0.5
(2,3)	0.5
(2,4)	0.5

A Naïve Solution

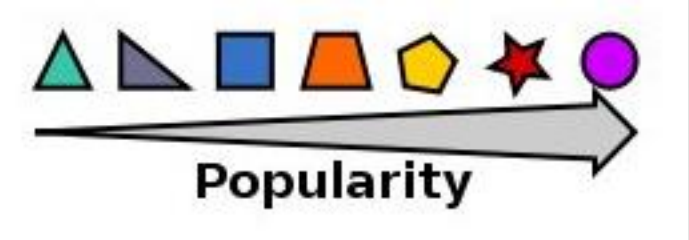
- Map: <23, (a,b,c)> → (a, 23), (b, 23), (c, 23)
- Reduce: (a,23),(a,29),(a,50), … → Verify each pair (23, 29), (23, 50), (29, 50) … …



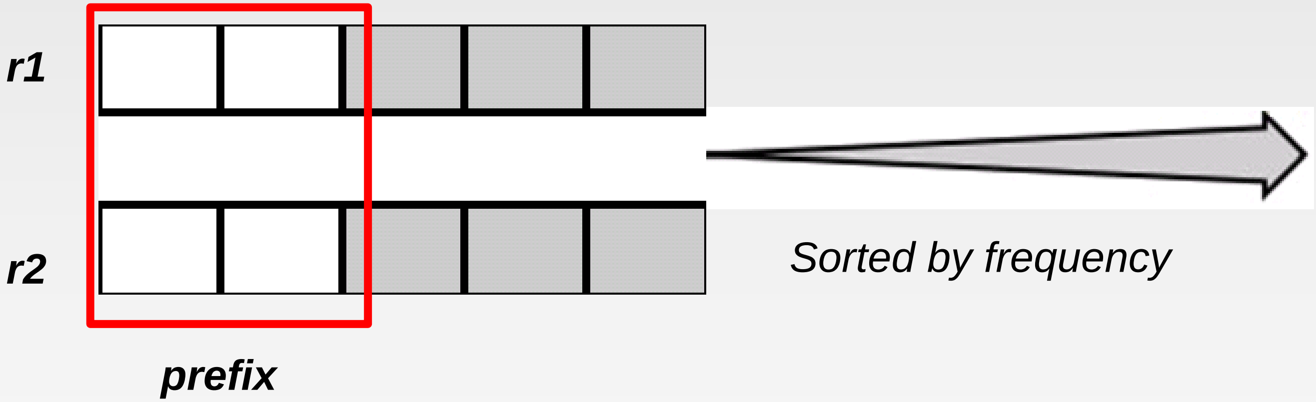
- Too much data to transfer ☹
- Too many pairs to verify ☹

Solving frequency skew: prefix filtering

- Sort tokens by frequency (ascending)

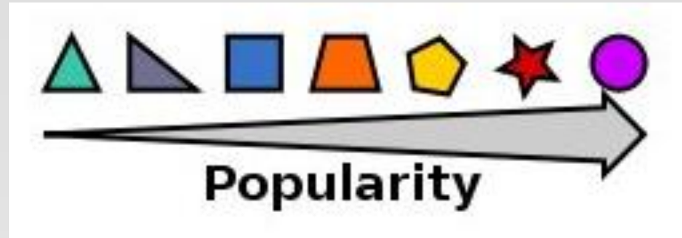


- Prefix of a set: least frequent tokens



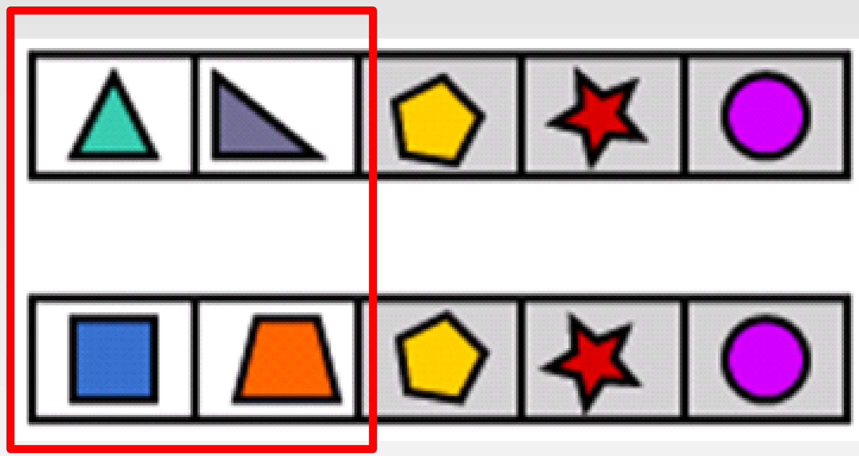
- Prefixes of similar sets should share tokens

Prefix filtering: example



Record 1

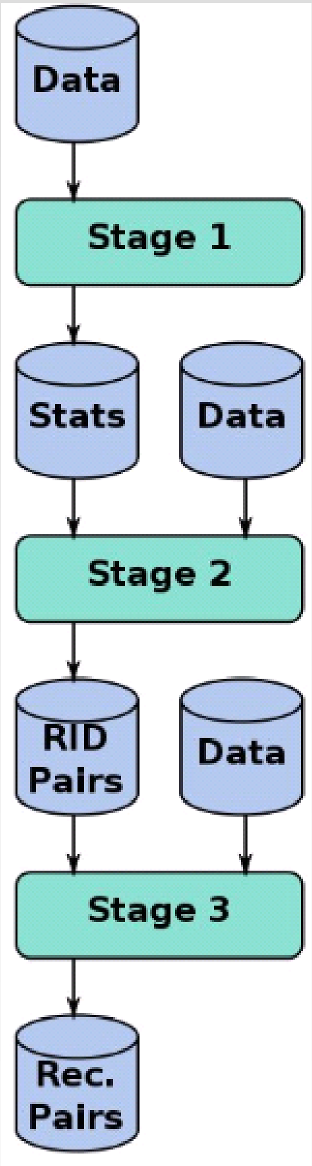
Record 2



- Each set has 5 tokens
- "Similar": they share at least 4 tokens
- Prefix length: 2

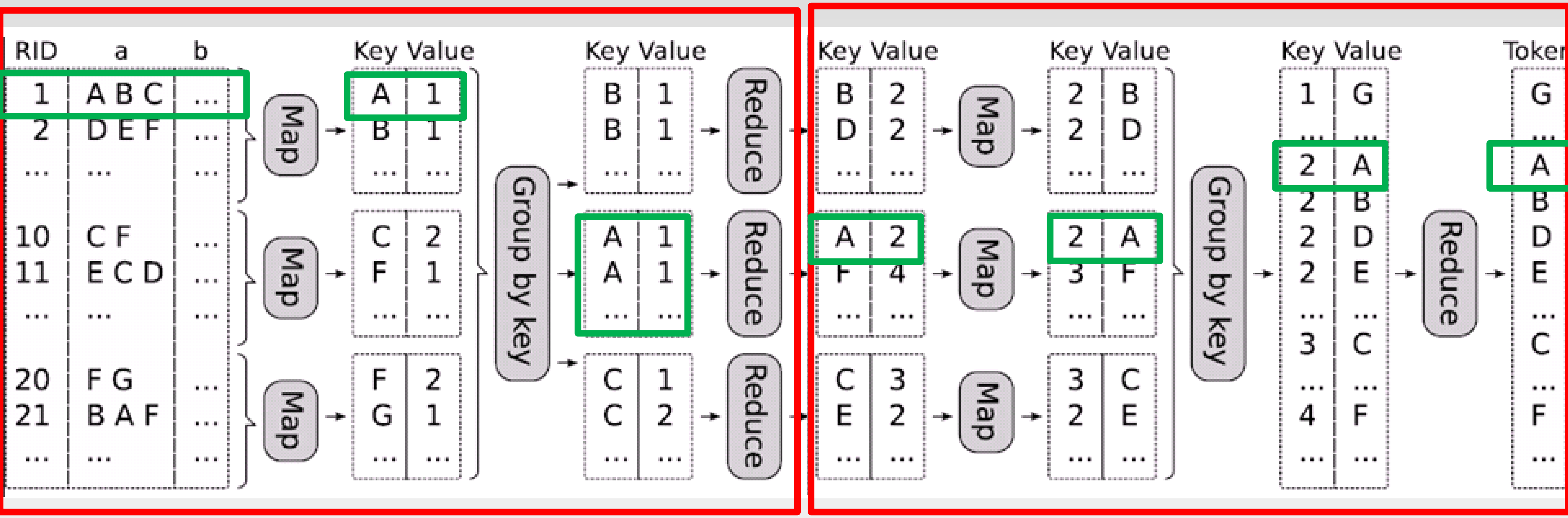
Hadoop Solution: Overview

- Stage 1: Order tokens by frequency
- Stage 2: Finding "similar" id pairs (verification)
- Stage 3: remove duplicates



Efficient Parallel Set-Similarity Joins Using MapReduce. SIGMOD'10

Stage 1: Sort tokens by frequency



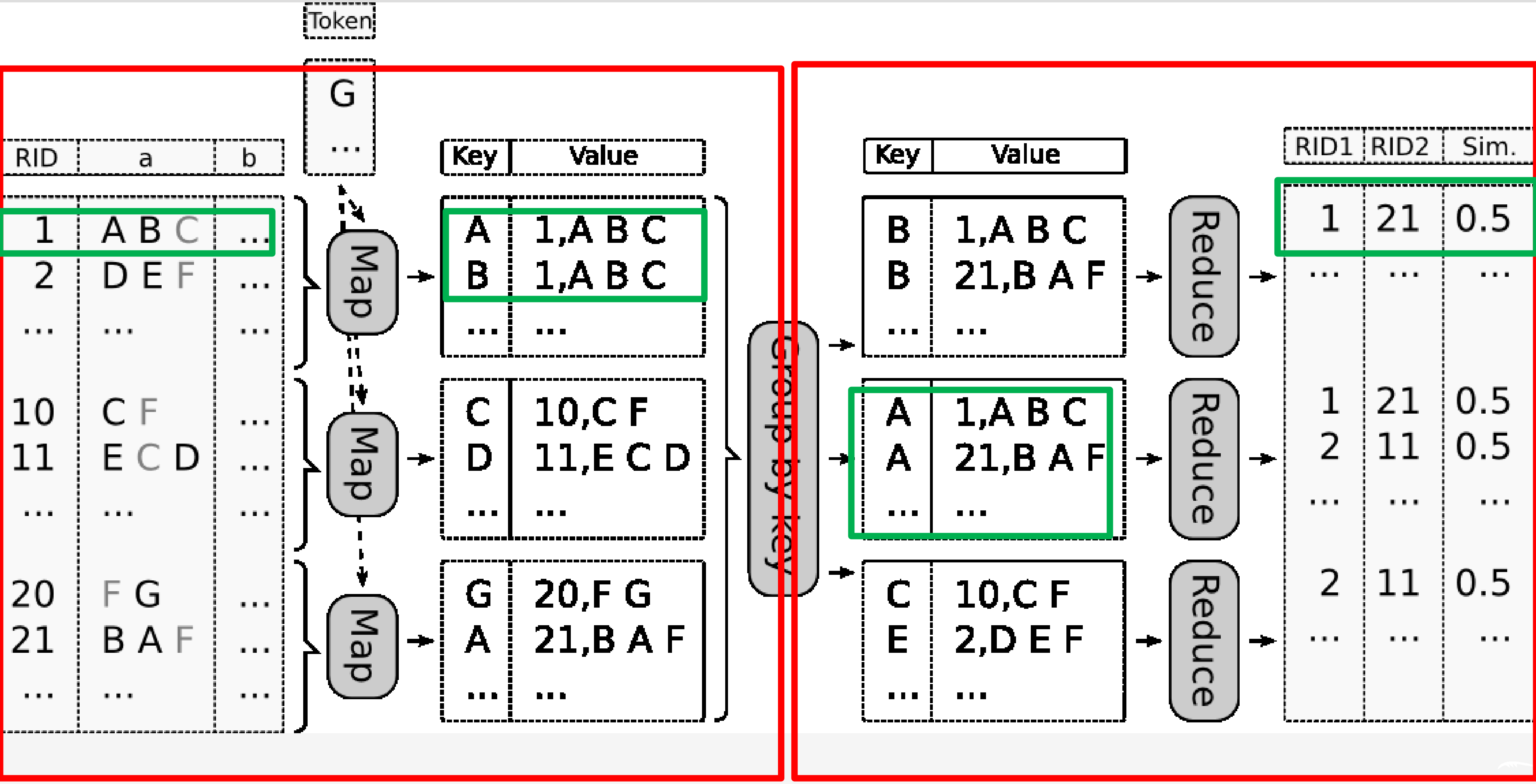
Compute token frequencies

Sort them

MapReduce phase 1

MapReduce phase 2

Stage 2: Find “similar” id pairs



Partition using prefixes

Verify similarity

Stage 3: Remove Duplicates

RID1	RID2	Sim.
1	21	0.5
…	…	…
1	21	0.5
2	11	0.5
…	…	…
2	11	0.5
…	…	…

Compute the Length of Shared Tokens

- Jaccard Similarity: **sim(r, s) = |r∩s|/|r∪s|**

- If **sim(r, s) >= τ, |r∩s| >= |r∪s| * τ >= max(|r|, |s|) * τ >= |r| * τ = l**

- Given a record r, you can compute the prefix length as p = |r| - l + 1

- r and s is a candidate pair, they must share at least one token in the first (|r| - l + 1) tokens

- Given a record r = (A, B, C, D) and p = 2, the mapper emits (A, r) and (B, r)

More Optimization Strategies

- **(Optional)** Do it using Spark on Google Dataproc
- It is your job to design more optimization strategies. The faster the better!
- Thinking:
  - How to compute the prefix length of a single record when processing it?
  - How to pass the sorted list to each worker?
  - Is it necessary to compute the similarity for duplicate pairs?

More Tips on Project 3

- 1. It is suggested to use Spark DataFrame APIs. It is also fine to use Spark RDD APIs.
- 2. You CANNOT use LSH to do this project, since that can only obtain approximate results. This project requires exact set similarity join results. Please follow the slides as introduced during the lecture.
- 3. One more test case will be released to you soon.
- 4. Some hints to accelerate your program:
  - Do not use string for storing the elements.
  - Please broadcast the frequency lookup table.
  - Try to avoid computing similarities for the duplicated pairs.
- 5. You do not need to merge the results.
- 6. To guarantee the fairness of comparing the efficiency, we will run your code in the VM using two local threads with Spark's default partition.

More Tips on Project 3 (Dataproc)

- 1. Check your bill!!! Be careful of how much you have already spent. Remember to terminate the cluster and delete the data in your bucket after you finish your jobs!!! If you have used the $300 credits, you can register with Dataproc using a new email.
- 2. If the CPU limit of your account is only 8, you just need to create two clusters: one with 2 worker nodes and the other one with 3 worker nodes. I do not know why the CPU limit is different for us, and I haven't found a solution for this.
- 3. Someone may see some error messages like: "Broadcasting large task binary with size XXXMB" or "java.lang.InterruptedException". If you job can complete successfully, you can ignore these messages. I also saw such messages when running my job.
- 4. Do not use SetMaster("local") when running on Dataproc.
- 5. More partitions of your RDD/DataFrame could be helpful on clusters with more worker nodes.

References

- Chapter 3 of Mining of Massive Datasets.

End of Chapter 7.2